Framework

(see *P & T*, chapter 8)

2 parties, 3 groups of voter
voters differ only in **ideology**, not income ; the same number of people of each type

so type–1 voters tend to vote left, and type 3 voters tend to vote right

the left party will tend to get the support of type–1 voters, and the right party will tend to get the support of type–3 voters

so that the support of the "swing" type–2 voters will be most important in deciding the election

Question

each party will try and attract the swing voters by making transfers to group 2

question : how do these transfers differ between **proportional representation** and a **constituency system** in which each constituency is inhabited by a different type of voter?

result : a constituency system results in (i) bigger transfers to the swing voters ; (ii) a smaller, more efficient public sector ; (iii) less revenue diversion by politicians, compared to proportional representation

The Mechanism

under proportional representation, for each party : every vote counts

so even though most type–1 voters support the left party, if the right party can shift its policies to make them a little more attractive to type–1 voters, they will gain some votes from these (type–1) voters

so parties are competing for the votes of all 3 types of voters under proportional representation (even though the left party will wind up with most of the type–1 votes and the right party will wind up with most of the type–3 votes)

under a constituency system...

with a constituency system, the left party is nearly certain to win constituency 1 [inhabited (mostly) by type–1 people] and the right party is nearly certain to win constituency 3

so neither party has a strong incentive, at the margin, to try and attract more votes from type–1 or type–3 voters : if the left party makes its policy slightly less attractive to type–1 voters, they are still (nearly) certain to win constituency 1, and if the right party makes its policy slightly more attractive to type–1 voters, they are still (nearly) certain to lose constituency 1

so a constituency system induces both parties to tailor their policies more to the crucial swing constituency, constituency 2

Probabilistic Voting

voter i in group J votes for the left party L if (and only if)

$$W^J(L) > W^J(R) + \delta + \phi^{iJ} \tag{1}$$

where δ is a common random term denoting the "general popularity" of the right party, and ϕ_{iJ} is an "idiosyncratic" term, reflecting the "personal popularity" of the right party with voter i of type J

and where $W^J(L)$ and $W^J(R)$ are the "non–random" levels of welfare a voter of type J gets from the policies of the two parties

Non–random Welfare

$$u^J = (1 - \tau) + f^J + H(g) \tag{2}$$

so that everyone has the same income 1, everyone pays the same tax τ, everyone consumes the same level g of public output — and every voter of type J gets the same targeted transfer f^J

the government budget constraint is

$$3\tau = g + f^1 + f^2 + f^3 + r \tag{3}$$

where r is the money diverted to politicians

so substitution of (3) into (2) implies that

$$W^J = 1 + \frac{2f^J}{3} - \frac{r}{3} - \frac{f^K}{3} - \frac{f^M}{3} + H(g) - \frac{g}{3} \tag{4}$$

where K and M label the other 2 groups

Random Components

the "general popularity" term δ is a random draw from the uniform distribution over $[-\frac{1}{2\Psi}, \frac{1}{2\Psi}]$

while the idiosyncratic random term ϕ^{iJ} is a random draw from a distribution which is uniform over

$$[-\frac{1}{2\Phi^J} + \bar{\sigma}^J, \frac{1}{2\Phi^J} + \bar{\sigma}^J] \tag{5}$$

where

$$\sigma^1 < 0 = \sigma^2 < \sigma^3 \tag{6}$$

(which is why type–1 voters tend to vote left and type–2 voters tend to vote right)
[also $\bar{\sigma}^1\Phi^1 = -\bar{\sigma}^3\Phi^3$]

vote shares

everyone of type J whose idiosyncratic parameter σ^{iJ} is low enough such that

$$W^J(\mathbf{q}_L) > W^R(\mathbf{q}_R) + \delta + \sigma^{iJ} \tag{7}$$

will vote for the left party ; everyone whose σ^{iJ} is higher than that will vote for the right party

where $\mathbf{q_n}$ is the policy $(g_n, f_n^1, f_n^2, f_n^3, r_n)$ chosen by party n

Vote Shares

the left party gets the votes of the type–J voters for whom $W^J(\mathbf{q}_L) > W^R(\mathbf{q}_R) + \delta + \sigma^{iJ}$

since σ^{iJ} is distributed uniformly over $[-\frac{1}{2\Phi^J} + \bar{\sigma}^J, \frac{1}{2\Phi^J} + \bar{\sigma}^J]$,

therefore the fraction π_L^J of the type–J vote going to the left party is

$$\pi_L^J = \Phi^J[W^J(\mathbf{q}_L) - W^J(\mathbf{q}_R) - \delta - \bar{\sigma}^J] + 0.5 \tag{8}$$

which is a random variable since the universal popularity parameter δ is random

Each Party's Goal

parties would like to divert money to themselves, but they also get a payoff from having power

party n ($n \in \{L, R\}$) chooses its policy $\mathbf{q}_n \equiv (g_n, f_n^1, f_n^2, f_n^3, r_n)$ so as to maximize

$$p_n(R + \gamma r_n)$$

where p_n is the probability it is elected (which depends on its own policies, and those of the other party

where R is the value the party places on staying in power, and γ measures the value of diverted money to the party (relative to the value of staying in power)

The Power of Group 2

it is assumed that — in addition to being in the middle — group 2 has less idiosyncratic variation than the other groups

ASSUMPTION : $\Phi^2 > \Phi^1 = \Phi_3$

the section on probabilistic voting showed that groups with high Φ^J had more power

here, equation (8) implies that increasing the transfer f_2^L to group 2 by ϵ will
increase π_2^L by $\frac{2}{3}\Phi^2\epsilon$ and
decrease π_1^L and π_3^L by $\frac{1}{3}\Phi_1\epsilon$ each

which must increase the left party's total vote share (among all 3 groups), because of the assumption above

Under Proportional Representation

each party's chance of winning is the probability that its total vote share $\pi_1^n + \pi_2^n + \pi_3^n$ exceeds 0.5

for the left party, that probability equals

$\alpha[\Phi^1 W^1(\mathbf{q}_L) + \Phi^2 W^2(\mathbf{q}_L) + \Phi_3 W^3(\mathbf{q}_L)] + \beta$

where α and β are constants

($\alpha = \frac{\Psi}{\Phi^1+\Phi^2+\Phi^3}$ and
$\beta = 0.5 - \alpha[\Phi^1 W^1(\mathbf{q}_R) + \Phi^2 W^2(\mathbf{q}_R) + \Phi_3 W^3(\mathbf{q}_R)]$)

The Power of Group 2

the assumption that $\Phi^2 > \Phi^1 = \Phi^3$ means that, for each party n,

$$f^2 > 0 = f^1 = f^3$$

it also means that each party wants to tax away all the income it can from voters — and transfer the proceeds to type–2 voters

increasing τ by ϵ, and using the money raised (from all 3 groups) to increase f^2 by 3ϵ will

decrease W_L^1 and W_L^3 by ϵ each, and increase W_L^2 by 2ϵ, which must result in an increase in
$\Phi^1 W^1(\mathbf{q}_L) + \Phi^2 W^2(\mathbf{q}_L) + \Phi_3 W^3(\mathbf{q}_L)$

Public Good Provision

since the tax rate is the maximum possible, increases in public good provision are financed by reductions in transfers f^2 to middle–income people

so that the effect on the left party's probability of winning, when it changes public good provision g, is proportional to

$$(\Phi^1 + \Phi^2 + \Phi^3)H'(g) - \Phi^2 \tag{9}$$

the left party's preferred public good promise g_L is the level which makes expression (9) equal 0, so that

$$1 > H'(g) = \frac{\Phi^2}{\Phi^1 + \Phi^2 + \Phi^3} > 1/3$$

meaning that public goods are under–provided compared to the efficient level g^* for which

$$H'(g^*) = 1/3$$

Diversion under Proportional Representation

the left party chooses its diversion r of funds so as to maximize

$$p_L(R + \gamma r)$$

leading to a first-order condition

$$-\frac{\partial p_L}{\partial r}(R + \gamma r) = 0.5\gamma \tag{10}$$

since the probability p_n of either party being elected in equilibrium is 0.5

increases in diversion r come from somewhere in the budget — from a decrease in f^2, which decreases $W^2(\mathbf{q}_L)$ by 1 — which means that the solution to equation (10) is

$$R + \gamma r = \frac{0.5\gamma}{\Phi^2 \alpha} \tag{11}$$

Constituencies

assumption : each type of voter lives in a separate district

so district 1 has all type–1 voters, district 2 all type–2 voters, district 3 all type–3 voters

if extreme voters' biases are big — $\bar{\sigma}^1$ is very negative and $\bar{\sigma}^3$ is very positive — then the left party wins district 1 and the right party wins district 3

so the election depends (entirely) on district 2

[these assumptions can be relaxed : what's crucial is that votes of type–2 voters have much more influence on the outcome of the election]

Parties' Maximization with Constituencies

with constituencies, the probability p_L that the left party wins the election is the probability that it gets more than 50% of the votes of the type–2 voters

$$p_L = \mathrm{Prob}[\pi_L^2 > 0.5] \tag{12}$$

so that (from equation(8)

$$p_L = \psi[W^2(\mathbf{q}_L) - W^2(\mathbf{q}_R)] + 0.5 \tag{13}$$

Taxation and Transfers

under the constituency system, it is still the case that each party

sets the maximal taxes : $\tau = 1$

and transfers money only to the swing voters :

$$f^1 = f^3 = 0 \qquad f^2 > 0$$

[why? if the left party raises its proposed τ_L, and uses the money to increase its proposed transfer f_L^2 to the swing voters, then the policy change will increase $W^2(\mathbf{q}_L)$, which increases its chances of winning the election

and if f_L^1 or f_L^3 were positive, then a slight decrease in f_L^1 or f_L^3, transferring the money to an increase in f_L^2, would increase $W_2(\mathbf{q}_L)$, which would increase the left party's chance of winning the election]

Public Good Provision with Constituencies

increasing g_L by some ϵ, and financing this increase by a decrease (of ϵ) in the transfer f_L^2, would increase the payoff to swing voters by

$$\Delta W^2(\mathbf{q}_L) = [H'(g) - 1]\epsilon$$

so that the level g of public good provision proposed by each party will be the level such that

$$H'(g) = 1$$

constituencies reduce public good provision, compared with proportional representation

Diversion by Politicians

every increase in r (money diverted by politicians) must reduce f^2 by an equal amount [since taxes are set at their maximum level]

so that $\frac{dW^2}{dr_L} = -1$, which means (from equation (13) that

$$\frac{\partial p_L}{\partial r_L} = -\Psi \tag{14}$$

the left party chooses r_L so as to maximize $p_L(R + \gamma r_L)$, leading to the first–order condition $-\Psi[R + \gamma r_L] + \gamma p_L = 0$, or

$$R + \gamma r = \frac{0.5\gamma}{\Psi} \tag{15}$$

Less Diversion

under proportional representation, the level of diversion chosen by each party satisfied equation (11),

$$R + \gamma r = \frac{0.5\gamma}{\Phi^2 \alpha}$$

, and under constituency representation, each party's diversion level satisfies equation (15),

$$R + \gamma r = \frac{0.5\gamma}{\Psi}$$

since

$$\Phi^2 \alpha = \frac{\Phi^2 \Psi}{\Phi^1 + \Phi^2 + \Phi^3} < \Psi$$

there will be **less** diversion under the constituency system than under proportional representation

Proportional Representation's Effects

in this model, the effects of replacing Canada's constituency system with proportional representation would be :

1. a higher level of public good provision
2. smaller transfers to the pivotal group (group 2)
3. more diversion of funds by politicians
4. better–off : fringe voters (groups 1 and 3), politicians
5. worse–off : voters in the pivotal group