

due : Wednesday October 10 before class

Do all 5 questions. Each counts 20%.

1. Explain why the following three equations cannot be the Marshallian demand functions of a consumer with well-behaved preferences, even when $p_1 \geq p_3$.

$$\begin{aligned} x_1(\mathbf{p}, y) &= \frac{y}{2p_1} \\ x_2(\mathbf{p}, y) &= \frac{p_3 y}{2p_1 p_2} \\ x_3(\mathbf{p}, y) &= \frac{(p_1 - p_3)y}{2p_1 p_3} \end{aligned}$$

2. Find all the violations of the strong and weak axioms of revealed preference in the following table, which indicates the prices p^t of three different commodities at three different times, and the quantities x^t of the 3 goods chosen at the three different times. (For example, the second row indicates that the consumer chose the bundle $\mathbf{x} = (24, 21, 20)$ when the price vector was $\mathbf{p} = (2, 1, 2)$.)

t	p_1^t	p_2^t	p_3^t	x_1^t	x_2^t	x_3^t
1	1	2	2	22	20	23
2	2	1	2	24	21	20
3	2	2	1	21	23	21

3. Frank and Ernest are both risk-averse expected utility maximizers. Frank has a utility-of-wealth function

$$U(W) = \ln W$$

while Ernest has a utility-of-wealth function

$$V(W) = 1 - e^{-W}$$

Give an example of a simple (“2 state”) gamble which Ernest would accept but which Frank would reject, and an example of another simple (“2 state”) gamble which Frank would accept but which Ernest would reject.

4. Suppose that an expected utility maximizer has a utility-of-wealth function

$$U(W) = \frac{1}{1-\beta} W^{1-\beta} \quad \beta < 1 \quad \beta \neq 0$$

The person has an initial wealth of W_0 . She has the opportunity to risk all her initial wealth on the toss of a fair coin. If the coin lands “tails”, she would lose all her wealth. If the coin lands “heads”, she would collect a multiple A of her initial wealth, where $A > 2$. (This is an “all or nothing” proposition. She must bet W_0 if she bets.)

For what values of initial wealth W_0 , and of the parameter β , would the person be willing to participate in this risky opportunity?

5. Suppose that there were two states of the world, good and bad, and three assets. Asset 1 pays a net return of 10 percent in either state of the world. Asset 2 pays a net return of 20 percent in the bad state, and 0 in the good state. Asset 3 pays a net return of r percent in the good state, and 0 in the bad state.

The probability of the good state is π and of the bad state is $1 - \pi$.

The person can allocate her wealth among the 3 assets, but must have a non-negative amount invested in each assets (that is, she is not allowed to sell an asset short).

If the person is a risk-averse expected utility maximizer, for what values of r will she choose to invest positive amounts in all 3 assets?