GS/ECON 5010 Assignment 3 F2008

due : Friday November 7 8:30 am

Do all 5 questions. Each counts 20%.

1. Find the profit function, supply function, and unconditional input demand functions for a firm with a production function

$$f(x_1, x_2) = 2\sqrt{x_1} + \ln(x_2 + 1)$$

(do not assume that w_2 must be less than p, where w_2 is the price of input 2, and p is the price of output)

2. What is the equation of the supply curve of a firm which has a long–run total cost function with the equation

$$TC(q) = 6q^2 - 36q + 216\frac{q}{q+1}$$

where q is the quantity of output produced by the firm?

(You do not need to solve for the equation of the supply curve in closed form, just the relation between price and output supplied).

3. Derive the aggregate supply curve for apples for the following imaginary area.

The area consists of the land on both sides of a river. On each side of the river, all the land within 1 km of the river can be used for agriculture. The land can be used for grape–growing or for apple–growing. The profit (net of all expenses) from growing grapes on land anywhere in the valley is 100 dollars per square kilometre of land.

If the land is used for apples, the apples must be shipped down the river to a port at the mouth of a river. It costs \$1 per km to transport a tonne of apples down the river. It costs \$10 to transport a tonne of apples to the (distant) market. (So the cost of transporting 1 tonne of apples to market will be $10 + z$, if the farm is z kilometres upriver.)

Apples are grown using labour as the only input. The number of tonnes of apples which can be produced (per square kilometre of land) is

$$q = 2\sqrt{L}$$

where L is the amount of labour used (per square kilometre). The wage rate of labour is \$1 per hour ; this wage rate is the same everywhere in the valley.

There are many small farms along the valley, each owned by a different farmer who can choose to use the land either for apple production or for grape production.

4. What is the Cournot–Nash equilibrium in a duopoly in which the inverse demand function for the homogeneous output of the 2 firms is

$$p = a - Q$$

where Q is aggregate output, p the market price and a a positive constant, if the total cost of production of firm i is

$$C_i(q_i) = c_i q_i$$

where c_1 and c_2 are positive constants (not necessarily equal to each other) and q_i is the output of firm i?

5. Suppose the unit costs of the 2 duopolists in question #4 above were determined randomly. Firm i's unit cost c_i was c_H with probability 0.5 and c_L with probability 0.5, where $c_H > c_L$. Each firm's cost was determined by an independent draw from the same distribution. The firms choose their output levels after learning both their own cost draw, and the cost draw of the other firm. (That is, they each know c_1 and c_2 when they make their output decisions.)

Let $\bar{c} \equiv (0.5)(c_L + c_H)$ be the expected value of each firm's unit costs.

Each firm is risk–neutral, and wishes to maximize the expected value of its equilibrium profit.

Would firm 1 rather have a certain unit cost of $\bar{c}$, or the random draw? Would firm 1 rather have its rival (firm 2) with a certain cost $\bar{c}$, or a random draw? Explain your answer.