

due : Wednesday November 9 2:30 pm

Do all 5 questions. Each counts 20%.

1. What are the profit function, the supply function, and the (unconditional) input demand functions for a perfectly competitive firm with a production function

$$f(x_1, x_2) = \sqrt{\frac{x_1 x_2}{x_1 + 1}} \quad ?$$

2. What is the lowest price p for which a firm in perfect competition will be willing to produce a positive level of output, if the firm's cost function has the equation

$$C(w_1, w_2, y) = (\sqrt{w_1} + \sqrt{w_2})^2 \frac{y}{y+1} + w_1 y^2 \quad ?$$

3. Suppose that a market consists of an equal number of two types of people. Type-1 people's preferences can be represented by the utility function

$$U^1(x, z) = x + z - \frac{1}{2}z^2$$

and type-2 people's preferences can be represented by the utility function

$$U^2(x, z) = x + Vz - \frac{1}{2}z^2$$

where x and z are consumption of 2 different goods, and where $V > 1$. Good x is produced by a perfectly competitive industry, at a price of 1.

Good z is produced by a monopoly, which has **zero** production costs.

What price p should the monopoly charge so as to maximize its profits, if it must charge the same price p to all customers?

4. Suppose now that the monopoly in question #3 can charge a **two-part tariff**. That is, it requires all customers to pay a flat fee F in order to buy any of the good at all, along with a price p per unit.

[So customers have a choice : they either pay the fee F , and are able to buy as much or as little as they want of good z at a price of p per unit. Or they can decide not to pay the fee F , in which case they cannot purchase any of good z .]

The monopoly must charge the same fee F to all customers. And it must charge the same unit price p to all customers. If all other data are exactly as in question #3 above, what fee F and what unit price p should the monopoly charge?

5. In a Cournot oligopoly, in which n identical firms each have a constant marginal cost of production c , and in which market demand is defined by the inverse demand function

$$p = A - \frac{1}{\beta} \left(\sum_{i=1}^n x_i \right)^\beta$$

($\beta \neq 0$) for what values of β will it be true that the total industry output, in a symmetric Cournot equilibrium, increases with the number n of firms?