GS/ECON 5010 Section "B" Assignment 3 F2012

due : Wednesday November 7 before class

Do all 5 questions. Each counts 20%.

1. What is the profit function, and the long–run supply function, for a perfectly competitive firm with a production function

$$f(x_1, x_2) = \ln x_1 + \ln x_2 - \ln (x_1 + x_2) \qquad ?$$

2. What is the equation of the long–run supply curve for a perfectly–competitive industry, in which each of the (many) identical firms has a long run total cost function

$$TC(q) = q^3 - 18q^2 + 111q$$

where q is the quantity of output produced by the firm?

3. Suppose that consumers' preferences could be represented by the utility function

$$u(x_1, x_2) = x_1 + Ax_2 - (0.5)(x_2)^2$$

where A is some positive constant.

Suppose as well that good 1 is provided competitively, at a price of 1.

Good 2 is provided by a monopoly. The monopoly is thinking of the following price policy : customers have to pay a flat fee F in order to be able to buy from the monopoly at all ; they then can buy as much or as little of the monopoly's output as they want, at a price of p per unit. (That is, customers must make an up–front payment of F in order to buy anything at all from the monopoly.)

(i) What is the highest fee F that the monopoly can charge a customer, as a function of the price p per unit which it is charging?

(ii) If the monopoly's production cost is a constant c per unit, what price p, and what fee F should it charge to maximize profits, if all consumers are identical?

4. Another model of duopoly is that of **von Stackelberg**, in which firms choose output levels **sequentially**. That is, firm 1 chooses its output. Firm 2 observes what output level firm 1 has chosen, and then chooses its own output level. What output levels would the 2 firms choose, if they behaved in this manner, if they both produced an identical product for which the market inverse demand function had the equation

$$p = 15 - (q_1 + q_2)$$

if each firm had a total cost function

$$TC = \begin{array}{rl} 1 + 3q_i & \text{if} \quad q_i > 0 \\ 0 & \text{if} \quad q_i = 0 \end{array}$$

where q_i is the output level of firm i? [That is, each firm has a fixed cost of 1, and marginal cost of 3, and the fixed cost can be avoided only if the firm produces nothing at all.]

5. What does the contract curve look like for a 2–person, 2–good exchange economy, with a total endowment of 20 units of good 1 and 20 units of good 2, if the preferences of the two people could be represented by the utility functions

$$u^1(x_1^1, x_2^1) = x_1^1 + 2x_2^1$$

$$u^2(x_1^2, x_2^2) = 10 - \frac{2}{x_1^2} - \frac{1}{x_2^2}$$

where x_j^i is person i's consumption of good j?