GS/ECON 5010 Section "B" Assignment 4 F2012

due : Wednesday November 28 before class

Do all 5 questions. Each counts 20%.

1. What are the allocations in the core of the following 3–person, 2–good economy?

Each of the three people regards the two goods as **perfect complements** : her preferences can be represented by the utility function $u(x_1^i, x_2^i) = \min(x_1^i, x_2^i)$.

The endowments of the three people are $\mathbf{e}^1 = (1, 0), \mathbf{e}^2 = (2, 0), \mathbf{e}^3 = (0, 3)$.

2. Show that the following allocation is **not** in the core, in the 20-person economy described below : $x^i = (9, 9)$ for i odd, and $x^i = (11, 11)$ for i even.

In the economy, each person's preferences can be represented by the utility function

$$u^i(x_1^i, x_2^i) = x_1^i x_2^i$$

The endowment vectors are $e^i = (20, 0)$ for i odd, and $e^i = (0, 20)$ for i even.

3. What is the competitive (Walrasian) equilibrium in an exchange economy in which there are 1 million people of type 1, and 1 million people of type 2, in which each type–1 person has an endowment vector $\mathbf{e}^1 = (3, 1)$, each type–2 person has an endowment of $\mathbf{e}^2 = (2, 2)$ and each person, of either type, has preferences which can be represented by the utility function

$$u^i(x_1^i, x_2^i) = x_1^i [x_2^i]^2 \qquad ?$$

4. Find all the Nash equilibria (in pure and mixed strategies) in the following strategic–form two–person game.

	a	b	c	d
A	$(0, 1)$	$(6, 2)$	$(0, 0)$	$(10, 1)$
B	$(2, 3)$	$(4, 5)$	$(1, 4)$	$(8, 10)$
C	$(1, 6)$	$(0, 4)$	$(0, 8)$	$(6, 8)$

over

5. Find the subgame perfect Nash equilibrium to the following 2–player game.

The game has several stages. The 2 players are the owners (player 1) and the hockey players (player 2). In stage 1, player 1 gets to propose **shares** (s_1, s_2) of the available revenue, which is \$1 billion initially. So s_1 is the share of the revenue which goes to player 1, and $s_2 \equiv 1 - s_1$ is the share which goes to player 2.

Player 2 moves next. Player 1 can "accept" the original proposal, in which case the game ends, with payoffs of s_1 times 1 billion dollars for player 1, and s_2 times 1 billion dollars for player 2. Or player 2 can "reject" the initial proposal, and counter–propose a different split (t_1, t_2) of the revenue. However, due to the delay caused by the bargaining, if player 2 rejects the initial proposal, the available revenue will have shrunk, from \$1 billion, to \$800 million.

If player 2 has rejected the initial proposal, and made a counter–proposal, then player 1 gets to move again. Player 1 can "accept" player 2's counter–proposal, in which case the game ends, with payoffs of t_1 times 800 million to player 1, and t_2 times 800 million to player 2. Or player 1 can "reject" the counter–proposal, and make a new (third) proposal (u_1, u_2) for a split of the revenue. But due to the delay caused by the extended bargaining, if player 1 rejects this counter–proposal, the available revenue will have shrunk, from \$800 million, to \$600 million.

If the first two proposals have been rejected, there is a final move to the game. Player 2 gets to choose whether to accept player 1's new proposal (u_1, u_2), or to reject it. If the proposal is accepted, the game ends, with payoffs of u_i times \$600 million to player i. But if this last proposal is rejected, the game still ends. If this last proposal is rejected, player 2 will still get \$200 million (from playing in the Kontinental Hockey League), but player 1 will get a zero payoff, because the season will be cancelled.