GS/ECON 5010 Section "B" Assignment 4 F2013

due : Wednesday November 27 before class

Do all 5 questions. Each counts 20%.

1. What does the contract curve look like for a 2–person, 2–good exchange economy, with a total endowment of E_1 units of good 1 and E_2 units of good 2, if the preferences of the two people could be represented by the utility functions

$$u^1(\mathbf{x}^1) = 100 - (x_1^1)^{-\beta+1} - (x_2^1)^{-\beta+1}$$

$$u^2(\mathbf{x}^2) = 100 - a(x_1^2)^{-\beta+1} - (x_2^2)^{-\beta+1}$$

where $a > 1$, $\beta > 1$, and x_j^i is person i's consumption of good j? [The superscripts in the definition of u^2 are the person's name, "2", not "squared".]

2. Find 3 allocations in the core of the following 3–person, 2–good economy.
Person i's preferences can be represented by the utility function $u^i(x_1^i, x_2^i)$, where

$$u^1(x_1^1, x_2^1) = x_1^1$$

$$u^2(x_1^2, x_2^2) = x_2^2$$

$$u^3(x_1^3, x_2^3) = x_1^3 x_2^3$$

and the endowment vectors of the three people are $\mathbf{e}^1 = \mathbf{e}^2 = \mathbf{e}^3 = (1, 1)$.

3. Find the competitive equilibrium to a 2–good, 3–million–person economy, in which each person's endowment is $\mathbf{e}^i = (1, 1)$, and which 1 million people have preferences like person 1 in the previous question (# 2), 1 million people have preferences like person 2 in the previous question, and 1 million people have preferences like person 3 in the previous question. [That is, find a competitive equilibrium to an economy which is the economy of question #2 cloned one million times.]

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4. Write down the strategic form for the game described below, and find all the Nash equilibria to it.

The two players in the game are governments of two regions. Each government is trying to attract a firm, which will build a factory in one of the regions. (The factory's owner is **not** a player in this game.)

The two governments are choosing (simultaneously) whether or not to offer a tax exemption to the factory. It is common knowledge that the factory will locate in the region offering the tax exemption, if only one region chooses to offer an exemption. If neither region offers an exemption — or if both regions offer an exemption — the factory owner will flip a coin, and locate in either region with probability 0.5.

Each government places a value of 100 million dollars on having the factory in its region. A government will also collect T million dollars in taxes from the factory — if the factory locates in the region, and if the region has **not** offered a tax exemption. If the region offers a tax exemption, it collects no tax revenue from the factory.

Governments are risk neutral, and seek to maximize their expected returns, from having a factory in their region and from any taxes collected from the factory.

5. Find all the pure–strategy Nash equilibria to the game described in the previous question, if there now are $N > 2$ different regions (and still only one factory), so that, if $I > 0$ regions offer a tax exemption, each of the I regions gets the factory with probability $1/I$, and if none of the regions offers a tax exemption then each of them gets the factory with probability $1/N$.