GS/ECON 5010 Assignment 2 F2017

due : Monday October 23 3:00 pm

Do all 5 questions. Each counts 20%.

1. Could the following three functions be Marshallian demand functions for a consumer with well–behaved preferences? Explain briefly.

$$x_1(\mathbf{p}, y) = \frac{y}{p_1} - \sqrt{\frac{p_2}{p_1}} - \sqrt{\frac{p_3}{p_1}}$$

$$x_2(\mathbf{p}, y) = \sqrt{\frac{p_1}{p_2}}$$

$$x_3(\mathbf{p}, y) = \sqrt{\frac{p_1}{p_3}}$$

2. Find all the violations of the strong and weak axioms of revealed preference in the following table, which indicates the prices $p^t$ of three different commodities at four different times, and the quantities $x^t$ of the 3 goods chosen at the four different times. (For example, the third row indicates that the consumer chose the bundle $\mathbf{x} = (25, 15, 10)$ when the price vector was $\mathbf{p} = (3, 1, 1)$.)

| $t$ | $p_1^t$ | $p_2^t$ | $p_3^t$ | $x_1^t$ | $x_2^t$ | $x_3^t$ |
|---|---|---|---|---|---|---|
| 1 | 1 | 1 | 3 | 20 | 20 | 10 |
| 2 | 2 | 1 | 2 | 30 | 10 | 20 |
| 3 | 3 | 1 | 1 | 25 | 15 | 10 |
| 4 | 1 | 3 | 1 | 20 | 15 | 20 |

3. If a risk–averse utility maximizer had a utility–of–wealth function

$$u(W) = \log W$$

what would be the risk premium to a gamble which would double her wealth with probability 0.5, and cut her wealth in half with probability 0.5?

4. Find a utility function $U(\cdot)$ such that the following statement would be true for an expected utility maximizer with this utility function : "If my initial wealth was 100, I would not make this investment, but if my initial wealth were 200 I would make the investment. The investment will decrease my wealth by 40 with probability 0.5, and increase my wealth by 50 with probability 0.5."

**question 5 is on the next page**

5. How would the tax rate $t$ on capital gains affect the following [risk–averse, von Neumann–Morganstern expected utility maximizing] person's behaviour?

The person has an initial wealth of $W$. She chooses how much of that wealth to invest in a stock. The stock will either succeed or fail.

If the stock succeeds, she will make a gain of $G$ dollars per dollar invested (so that if she had invested $x$ dollars in the stock, the investment would now be worth $(G+1)x$ dollars.)

If the stock fails, it is worthless.

The probability of success for the stock is $\pi$.

Any money which she does **not** invest in the stock earns a net return of zero. (So that the $W - x$ dollars she does not invest in the stock will still be worth $W - x$, for certain.)

She must pay a tax of $t$ dollars for every dollar of capital gains, but gets a tax credit of $t$ dollars for every dollar she loses in the stock market.