due : Monday November 13   3.00 pm

Do all 5 questions. Each counts 20%.

1. For what input levels (x_1, x_2, x_3) does the following production function exhibit increasing returns to scale (using the "local" measure $\mu(\mathbf{x})$ of scale economies)?

$$f(\mathbf{x}) \equiv [x_1 x_2]^\alpha \frac{x_3}{1 + x_3}$$

where $\alpha > 0$.

2. Derive the cost function for the production function

$$f(x_1, x_2) = \log(x_1 + 1) + \log(x_2 + 1) - \log(x_1 + x_2 + 2) + \log 2$$

3. What are total industry profits (as a function of the price p in long–run equilibrium in the following perfectly competitive industry?

A type–t firm has a total cost function, as a function of the firm's output q of

$$C_i(q) = \frac{1}{2}q^2 + At$$

where A is a positive constant.

There is a continuum of firms. Firms differ in their type t : this type is distributed uniformly over $[0, 1]$. The total "number" of firms (that is the measure of all firms of all types) is some finite M.

[So half the firms have a value of t between 0 and 0.5, a quarter of the firms have a value of t between 0 and 0.25, etc.]

(questions #4 and #5 on the next page)

4. What is the equilibrium when two firms choose quantities simultaneously (that is, in a Cournot duopoly) if the market demand function for the firms' identical products was

$$p = A - Q$$

where $Q \equiv q_1 + q_2$ is the combined output of the two firms, when firm 1's total cost function is

$$TC_1(q_1) = q_1$$

and firm 2's total cost function is

$$TC_2(q_2) = q_2 + 4 \quad \text{if} \quad q_2 > 0 \quad ; \quad TC_2(0) = 0$$

where A is some positive constant? [So firm 2 has a fixed cost of 4, which it can avoid only by producing nothing.]

5. Find a symmetric Bertrand equilibrium, when two firms produce goods which are close (but imperfect) substitutes, with each firm i facing a demand curve

$$q_i = \frac{p_i^{-\alpha-1}}{p_i^{-\alpha} + p_j^{-\alpha}}$$

where p_j is the other firm's price, when each firm has a constant marginal cost c.

The parameter α is positive (and equals $\sigma - 1$, where σ is buyers' elasticity of substitution between the 2 goods).