GS/ECON 5010 Assignment 4 F2017

due : Monday November 27 before class

Do all 5 questions. Each counts 20%.

1. What does the contract curve look like for a 2–person exchange economy, in which the preferences of the two people can be represented by the utility functions

$$U^1(x_1^1, x_2^1) = \log(x_1^1) + x_2^1$$

$$U^2(x_1^2, x_2^2) = \log(x_1^2) + \log(x_2^2) \qquad ?$$

2. Show that the following allocation is not in the core of the 20–person exchange economy described below. (That is, find a coalition which **blocks** the allocation.)

The allocation is

$$\mathbf{x}^i = (76, 76) \quad \text{for} \quad i = 1, 2, 3, \cdots, 9$$

$$\mathbf{x}^{10} = (66, 66)$$

$$\mathbf{x}^i = (125, 125) \quad \text{for} \quad i = 11, 12, \cdots, 20$$

In this economy, the preferences of each of the 20 people can be represented by the utility function

$$u^i(x_1^i, x_2^i) = \log(x_1^i) + \log(x_2^i)$$

and the endowments are

$$\mathbf{e}^i = (150, 0) \quad \text{for} \quad i = 1, 2, \cdots, 10$$

$$\mathbf{e}^i = (50, 200) \quad \text{for} \quad i = 11, 12, \cdots, 20$$

3. Find all the allocations in the **core** of the following 3–person economy.

Each person has the same preferences : person i's preferences can be represented by the utility function

$$u^i(x_1^i, x_2^i) = x_1^i x_2^i \qquad i = 1, 2, 3$$

The endowment vectors $\mathbf{e}^i$ of the three people are

$$\mathbf{e}^1 = (3, 0)$$

$$\mathbf{e}^2 = (0, 3)$$

$$\mathbf{e}^3 = (1, 1)$$

4. Find a competitive equilibrium price vector for the following exchange economy.

There are 3 million people in the economy.

Each of the three million people has the same endowment vector,

$$\mathbf{e}^i = (e_1, e_2, e_3)$$

One million people are "type 1" people, and have preferences represented by the utility function

$$u^i(\mathbf{x}^i) = x_1^i x_2^i x_3^i$$

One million people are "type 2" people, and have preferences represented by the utility function

$$u^i(\mathbf{x}^i) = x_2^i$$

One million people are "type 3" people, and have preferences represented by the utility function

$$u^i(\mathbf{x})^i = (x_1^i)[(x_3^i)^2]$$

5. Find all the Nash equilibria (in pure and mixed strategies) to the following two–person game in strategic form.

	L	M	R
a	$(2,2)$	$(10,1)$	$(2,6)$
b	$(6,4)$	$(12,3)$	$(2,12)$
c	$(0,12)$	$(10,10)$	$(1,10)$
d	$(12,2)$	$(6,0)$	$(0,0)$