CES : Indirect Utility

Marshallian Demands

$$x_i(\mathbf{p}, y) = \frac{p_i^{r-1} y}{\sum_{j=1}^{n} p_j^r} \qquad (1)$$

Direct Utility Function

$$u(\mathbf{x}) = (\sum_{i=1}^{n} x_i^{\rho})^{1/\rho} \qquad (2)$$

substitute (1) into (2) to get

$$v(\mathbf{p}, y) = [\sum_{j=1}^{n} p_i^r]^{-1} [\sum_{i=1}^{n} p_i^{(r-1)(\rho)}]^{1/\rho} y \qquad (3)$$

rather conveniently,

$$(r-1)\rho = \rho(\frac{\rho}{\rho-1} - \frac{\rho-1}{\rho-1}) = \rho\frac{1}{\rho-1} = r \qquad (4)$$

so that (3) becomes

$$v(\mathbf{p}, y) = [\sum_{i=1}^{n} p_i^r]^{-1}[\sum_{i=1}^{n} p_i^r]^{1/\rho} y = [\sum_{i=1}^{n} p_i^r]^{1/\rho - 1} y \qquad (5)$$

which means that

$$v(\mathbf{p}, y) = [\sum_{i=1}^{n} p_i^r]^{-1/r} y \qquad (6)$$

which is the equation in the middle of page 32 in the textbook