## CES : Indirect Utility

Marshallian Demands

$$
\begin{equation*}
x_{i}(\mathbf{p}, y)=\frac{p_{i}^{r-1} y}{\sum_{j=1}^{n} p_{j}^{r}} \tag{1}
\end{equation*}
$$

## Direct Utility Function

$$
\begin{equation*}
u(\mathbf{x})=\left(\sum_{i=1}^{n} x_{i}^{\rho}\right)^{1 / \rho} \tag{2}
\end{equation*}
$$

substitute (1) into (2) to get

$$
\begin{equation*}
v(\mathbf{p}, y)=\left[\sum_{j=1}^{n} p_{i}^{r}\right]^{-1}\left[\sum_{i=1}^{n} p_{i}^{(r-1)(\rho)}\right]^{1 / \rho} y \tag{3}
\end{equation*}
$$

rather conveniently,

$$
\begin{equation*}
(r-1) \rho=\rho\left(\frac{\rho}{\rho-1}-\frac{\rho-1}{\rho-1}\right)=\rho \frac{1}{\rho-1}=r \tag{4}
\end{equation*}
$$

so that (3) becomes

$$
\begin{equation*}
v(\mathbf{p}, y)=\left[\sum_{i=1}^{n} p_{i}^{r}\right]^{-1}\left[\sum_{i=1}^{n} p_{i}^{r}\right]^{1 / \rho} y=\left[\sum_{i=1}^{n} p_{i}^{r}\right]^{1 / \rho-1} y \tag{5}
\end{equation*}
$$

$$
\begin{equation*}
v(\mathbf{p}, y)=\left[\sum_{i=1}^{n} p_{i}^{r}\right]^{-1 / r} y \tag{6}
\end{equation*}
$$

which is the equation in the middle of page 32 in the textbook

