

**time=2.5 hours**

Do any **6** of the following 10 questions. All count equally.

1. If a person's preferences can be represented by the utility function

$$u(x_1, x_2) = x_1 + 2\sqrt{x_2}$$

find the person's Marshallian demand functions for each good, her indirect utility function, her Hicksian demand function, and her expenditure function.

2. If a person had a fixed amount of wealth  $W_0$  to allocate between a safe asset and a risky asset, how would her expected-utility-maximizing portfolio vary with her wealth  $W_0$ , and with the parameter  $\alpha$ , if her utility of wealth could be written

$$U(W) = \frac{1}{1-\alpha} W^{1-\alpha}$$

where  $\alpha$  is a positive parameter?

3. If a firm's production function were homogeneous of degree  $h$ , for some  $h > 0$ , what form would the firm's long run total cost function take? Explain briefly.

**continued**

4. What would be the equilibrium price and quantity in the long run, in a competitive industry in which there were many identical firms, each with the same long run total cost function

$$TC(q) = q^3 - 12q^2 + 60q$$

where  $q$  was the output of the firm, if the market demand curve for the output of the firms had the equation

$$Q = 540 - 5p$$

where  $Q$  was the total quantity demanded, and  $p$  the price of the good?

5. What would be the equilibrium output in a Cournot duopoly, if there were a price ceiling  $\bar{p}$  imposed on the market?

6. What is the contract curve in the following two-person, two-good exchange economy?

Person 1's preferences can be represented by the utility function

$$U^1(x_1^1, y_1^1) = x_1^1 + \ln x_2^1$$

and person 2's by the utility function

$$U^2(x_1^2, x_2^2) = x_1^2 x_2^2$$

where  $x_j^h$  is person  $h$ 's consumption of good  $j$ . The total endowment of goods is 7 units of good 1, and 4 of good 2.

7. Show that every competitive equilibrium allocation (Walrasian equilibrium allocation) in an exchange economy must be in the core.

**continued**

8. Write down a game in strategic form, which has a Nash equilibrium in which some player chooses a weakly dominated strategy. Does the game have any other Nash equilibria?

9. Write down the strategic form of the following game, a slightly modified version of Bertrand's model, and find all its Nash equilibria :

— two firms produce an identical good, for which the market demand curve can be written  $Q = 3 - p$ , where  $Q$  is quantity demanded, and  $p$  is the price ; each firm's total cost of producing  $q$  units is  $q$

— firms choose prices simultaneously ; firms wish to maximize profits

— buyers buy from the lowest-price firm ; if both firms charge the same price, then each firm serves half the market

— each firm's price must be a non-negative **integer** less than or equal to 3

10. (a) Write down the extensive form of the following signalling game :

— the two players are a worker, and a firm

— the wage is **fixed** at 2 ; the output of an uneducated worker of type  $a$  is worth  $a$  to the firm ; the output of an educated worker of type  $a$  is worth  $a + 1$

— the worker's ability level  $a$  can be either 1 or 2 ; the probability of either ability level is  $1/2$  ; the worker knows her own ability, but the firm cannot observe the ability directly

— the worker moves before the firm ; she chooses whether or not to get an education ; the cost of education to her is  $2/a$ , if her ability is  $a$

— the firm observes whether or not the worker got educated ; he then chooses whether or not to offer a job (at a fixed wage 2) to the worker

(b) Show that there is a sequential (perfect Bayesian) equilibrium to this game, in which high-ability workers choose to get a diploma, and low-ability workers do not.