YORK UNIVERSITY   Faculty of Graduate Studies

Final Examination   December 15, 2012

Economics 5010 BF3.0 : Applied Microeconomics   S. Bucovetsky

time=2.5 hours

Do any **6** of the following 10 questions. All count equally.

1. What are the Marshallian demand functions for a person whose indirect utility function is

$$v(y, p_1, p_2) = \sqrt{2\ln(y) - \ln(p_1) - \ln(p_2) - 2\ln(2)}$$

(where "ln" refers to the natural logarithm)?

2. Suppose that a person is a von–Neumann–Morgenstern expected utility maximizer, with a **constant coefficient** β **of relative risk aversion**.

Suppose that there is some gamble $g \equiv (\pi_1 \circ a_1, \pi_2 \circ a_2, \cdots, \pi_N \circ a_N)$, where the a_i's are amounts of money, and the π_i's are probabilities (which sum to 1).

If every payoff to this gamble increases by the same proportion S, show that the person's risk premium for this gamble must go up proportionally to S. [That is, if $g' \equiv (\pi_1 \circ a_1 S, \pi_2 \circ a_2 S, \cdots, \pi_N \circ a_N S)$, then the risk premium for gamble g' must be S times the risk premium for gamble g.]

3. Prove that a firm's conditional demand for an input cannot be an increasing function of the price of that input.

continued

4. What is the equation of the long–run industry supply curve of a perfectly competitive industry in which there are a large number of identical firms, each of which has the same total cost function

$$TC(y) = 5y^3 - 60y^2 + 250y$$

where $TC(y)$ is the total cost of producing y units of output?

5. What is the Cournot–Nash equilibrium in which the two firms produce the same good, for which the quantity demanded is

$$Q = A - p$$

where Q is the total quantity demanded, p the price of the good, and A a positive constant, if the two firms have total costs of production

$$C_1(q_1) = c_1 q_1$$

$$C_2(q_2) = c_2 q_2$$

where q_i is the quantity produced by firm i, $C_i(q_i)$ the firm's total cost, and c_1 and c_2 are positive constants for which $c_1 > c_2$?

6. Is the allocation $\mathbf{x}^1 = (1.6, 1.6), \mathbf{x}^2 = (1.6, 1.6), \mathbf{x}^3 = (2.8, 2.8)$ in the core of the 3–person exchange economy described below? Explain why or why not.

Each of the 3 people have preferences which can be represented by the utility function $u(x_1^i, x_2^i) = 10 - \frac{1}{x_1^i} - \frac{1}{x_2^i}$. The endowment vectors of the three people are $\mathbf{e}^1 = (1, 3), \mathbf{e}^2 = (1, 3), \mathbf{e}^3 = (4, 0)$.

continued

7. Prove the first fundamental theorem of welfare economics for an exchange economy, that every competitive equilibrium allocation is Pareto optimal.

8. What are all the Nash equilibria (in pure and mixed strategies) to the following game in strategic form?

	a	b	c	d
I	$(3, 6)$	$(4, -2)$	$(0, 0)$	$(-1, -4)$
II	$(1, 10)$	$(2, 8)$	$(-2, 4)$	$(-3, 6)$
III	$(0, 0)$	$(6, 0)$	$(6, 3)$	$(2, 2)$
IV	$(-2, 4)$	$(5, 7)$	$(3, 4)$	$(5, 1)$

9. What will be the expected revenue from an English (oral ascending–bid) auction, if there are two bidders, and each bidder's private valuation of the object is an independent draw : each bidder values the object at \$16 with probability 0.25 and each bidder values the object at \$32 with probability 0.75?

continued

10. (a) Write down the extensive form representation of the following game of incomplete information.

Firm 1's marginal cost of production, c_1 is either 0 or 2. Firm 2 does not observe firm 1's marginal cost, but thinks that either outcome ($c_1 = 0$ or $c_1 = 2$) is equally likely. Firm 1 knows its own marginal cost.

Firm 2's marginal cost of production is $c_2 = 1$. Both firms know this.

The two firms produce the same good. Total quantity demanded for this good is $Q = 4 - p$, where p is the lowest price in the market.

Buyers will buy (only) from the lower–cost firm, if the firms choose different prices. If the firms set the same prices as each other, then they split the market.

Firm 1 moves first, after learning its own marginal cost. It chooses whether or not to enter the market. Entering the market is costly : the entry cost is 1.

Firm 2 is already in the market (and so does not have to pay an entry cost). After observing whether firm 1 entered, firm 2 chooses its price. Its price p_2 must be an integer : either $p_2 = 2$ or $p_2 = 3$. (If firm 1 chose, initially, not to enter, then this will be the end of the game.)

Firm 1, if it chose to enter intially, observes firm 2's price, and then sets its own price p_1, which also must be an integer, but can be 1, 2 or 3.

Then buyers buy, and the game ends. A firm's payoff is its profit (sales revenue minus costs of production), minus any entry costs (if it's firm 1, and it chose to enter).

(b) Show that there is a perfect Bayesian equilibrium (or a sequential equilibrium) to this game, in which firm 2 thinks that firm 1 will enter if and only if $c_1 = 0$, and firm 2's belief is correct.