GS/ECON 5010 section "B" answers to midterm exam

October 2015

$Q1$. State and prove the Slutsky equation, relating the derivatives of the Marshallian and Hicksian demand functions.

$A1$. The definitions of the Hicksian and Marshallian demand functions imply that

$$\mathbf{x}^M(\mathbf{p}, e(\mathbf{p}, u)) = \mathbf{x}^H(\mathbf{p}, u) \tag{1-1}$$

where $e(\mathbf{p}, u)$ is the expenditure function.

Differentiating the above expression with respect to some price p_j,

$$\frac{\partial x_i^M}{\partial p_j} + \frac{\partial x_i^M}{\partial y}\frac{\partial e(\mathbf{p}, u)}{\partial p_j} = \frac{\partial x_i^H}{\partial p_j} \tag{1-2}$$

for any good i.

Shepherd's Lemma says that

$$\frac{\partial e(\mathbf{p}, u)}{\partial p_j} = x_j^H(\mathbf{p}, u) \tag{1-3}$$

and the definition of Marshallian and Hicksian demand functions says that

$$x_j^H(\mathbf{p}, u) = x_j^M(\mathbf{p}, y) \tag{1-4}$$

when $y = e(\mathbf{p}, u)$. So $(1-3)$ and $(1-4)$ imply that $(1-2)$ can be written

$$\frac{\partial x_i^M}{\partial p_j} + \frac{\partial x_i^M}{\partial y}x_j^M(\mathbf{p}, y) = \frac{\partial x_i^H}{\partial p_j} \tag{1-5}$$

Moving the second term on the left side of $(1-5)$ to the right gives us the Slutsky equation

$$\frac{\partial x_i^M}{\partial p_j} = \frac{\partial x_i^H}{\partial p_j} - \frac{\partial x_i^M}{\partial y}x_j^M(\mathbf{p}, y) \tag{1-6}$$

$Q2$. If a person has a constant coefficient of relative risk aversion equal to 2, and initial wealth $2X$, what would be the highest amount that she would be willing to pay to insure completely against an accident causing a loss of X if she perceived the probability of that loss as equalling some π (with $0 < \pi < 1$)?

*A*2. (This is question #3 from Assignment 2.)

The person's von–Neumann–Morgenstern utility–of–wealth function is

$$U(W) = -W^{-1}$$

if she has a constant coefficient of relative risk aversion of 2. Her alternatives are to purchase the complete insurance against the loss at some total price P, leaving her with the loss (of X) fully covered in the "bad" state, and with wealth of $2X - P$ in either state of the world, giving her expected utility of

$$EU_I = -(2X - P)^{-1} \qquad (4-1)$$

or doing without any insurance, giving her an expected utility of

$$EU_N = -(1-\pi)(2X)^{-1} - \pi(X^{-1} \qquad (4-2)$$

If she is just willing to purchase the insurance, she should be indifferent between these alternatives. Setting expression $(4-1)$ equal to expression $(4-2)$, the maximum price P which she is willing to pay satisfies the equation

$$\frac{1}{2X - P} = \frac{1-\pi}{2X} + \frac{\pi}{X} \qquad (4-3)$$

which implies that

$$P = 2\frac{\pi}{1+\pi}X \qquad (4-4)$$

Notice, as expected, that the price she is willing to pay is proportional to her wealth (since she has a CRR von Neumann–Morgenstern utility–of–wealth function), and that the price she is willing to pay for insurance exceeds the expected loss πX.

*Q*3. What is the cost function for a firm with production function

$$f(x_1, x_2) = x_1 + \log(x_2 + 1) \qquad ?$$

*A*3. The first–order conditions for the minimization of $w_1x_1 + w_2x_2 + w_3x_3$ subject to $x_1 + \log(x_2 + 1) + \log(x_3 + 1) = y$ are

$$\mu = w_1 \tag{3 − 1}$$

$$\frac{\mu}{x_2 + 1} = w_2 \tag{3 − 2}$$

Substitution of (3 − 1) into (3 − 2) yields the conditional input demand functions for input 2 :

$$x_2(\mathbf{w}, y) = \frac{w_1}{w_2} - 1 \tag{3 − 3}$$

Since

$$y = x_1 + \log(x_2 + 1)$$

therefore

$$x_1(\mathbf{w}, y) = y - \log(x_2 + 1) = y - \log w_1 + \log w_2 \tag{3 − 4}$$

So the cost function, which equals $w_1x_1(\mathbf{w}, y) + w_2x_2(\mathbf{w}, y)$ is

$$C(\mathbf{w}, y) = w_1y + w_1 \log(\frac{w_2}{w_1}) + w_1 - w_2 \tag{3 − 5}$$

[The above results are true only if the firm's cost minimization involves use of both inputs. If the value of x_2 defined in equation (3 − 3) is negative, then we have a corner solution. So the analysis above applies only if $w_1 \geq w_2$.

If $w_1 < w_2$, then $x_2 = 0$, and $x_1 = y$, so that

$$C(w_1, w_2, y) = w_1y \tag{3 − 6}$$

when $w_1 < w_2$.

And, if $w_1 \geq w_2$, the value of x_1 defined by equation (3 − 4) will be negative if y is too small : so when $w_2 < w_1$ and $y < \log(\frac{w_1}{w_2})$, then $x_1 = 0$, with $x_2 = e^y - 1$ and $C(w_1, w_2, y) = w_2(e^y - 1)$.]