

**time=2.5 hours**

Do any **6** of the following 10 questions. All count equally.

1. State and prove **Roy's Identity**, describing the relation among the derivatives of the indirect utility function.

2. If a risk-averse expected utility maximizer had a utility-of-wealth function

$$U(W) = \log W$$

how much insurance coverage  $I$  would she buy against a fire causing damage  $L$ , if the probability of the fire were  $\pi$ , her initial wealth was  $W > L$ , and she could buy as much or as little insurance coverage  $I$  as she wished, at a price of  $p$  dollars per dollar of insurance coverage, with  $p \geq \pi$ ?

3. What is the cost function  $C(w_1, w_2, w_3, y)$  for a firm with a production function

$$y = f(x_1, x_2, x_3) = \min(\sqrt{x_1 x_2}, x_3) \quad ?$$

**continued**

4. What are the profit-maximizing prices that an smartphone monopoly should charge to Canadian and Chinese customers in the following situation?

The firm can produce smartphones at a constant marginal cost of \$100 each, in China. It also costs \$100 to ship a smartphone from China to Canada. (It is costless to ship to the Chinese market.) Aggregate consumer demand for the smartphone in Canada is

$$q = \left[\frac{p}{300}\right]^{-2}$$

where  $p$  is the price (in dollars), and  $q$  is the quantity sold (in millions). Aggregate demand in China is

$$Q = \left[\frac{P}{10}\right]^{-5}$$

where  $Q$  is the quantity sold (in millions) in China, and  $P$  is the price charged to Chinese consumers. [Resale from one market to the other is sufficiently difficult that the firm can charge separate prices in the two countries.]

5. What are the equilibria when two firms choose quantities simultaneously (that is, in a Cournot duopoly) if the market demand function for the firms' identical products was

$$p = 8 - Q$$

where  $Q \equiv q_1 + q_2$  is the combined output of the two firms, when firm 1's total cost function is

$$TC_1(q_1) = q_1$$

and firm 2's total cost function is

$$TC_2(q_2) = q_2 + 4 \quad \text{if } q_2 > 0 \quad ; \quad TC_2(0) = 0 \quad ?$$

[So firm 2 has a fixed cost of 4, which it can avoid only by producing nothing.]

**continued**

6. Find an allocation which is in the core of the following 4–person, 2–good exchange economy.

Person 1 and person 2 each have endowment vectors  $(3, 3)$  and person 3 and person 4 each have endowment vectors  $(6, 0)$ .

All 4 people’s preferences can be represented by the utility function

$$U(x_1^i, x_2^i) = x_1^i x_2^i$$

7. State and prove Walras’s Law, describing consumers’ excess demand functions in an exchange economy.

8. Find all the Nash equilibria, in pure and mixed strategies, to the following game in strategic form :

	<i>A</i>	<i>B</i>	<i>C</i>	<i>D</i>
<i>a</i>	(1, 8)	(5, 20)	(10, 2)	(0, 12)
<i>b</i>	(12, 3)	(20, 16)	(8, 16)	(4, 4)
<i>c</i>	(2, 0)	(12, 6)	(12, 2)	(4, 8)
<i>d</i>	(8, 4)	(20, 2)	(10, 6)	(8, 4)

**continued**

9. Write down the extensive form for the following game of incomplete information, and find a perfect Bayesian equilibrium (or sequential equilibrium) to it.

Farmer Jones owes Farmer Smith \$1000. Farmer Jones owns a horse. The value of the horse is random : with probability 0.2 the horse is worth \$3000, and with probability 0.8 the horse is worth 0. Farmer Smith knows these probabilities, but does not know the actual value of the horse. Farmer Jones knows the actual value of the horse. [That is : “nature” chooses whether the horse is worth \$3000, or worthless ; Farmer Jones observes nature’s move ; Farmer Smith does not observe nature’s move.]

Farmer Jones can choose whether or not to offer Farmer Smith the horse as a way of paying back the loan.

Farmer Smith can choose whether or not to accept this offer.

That is, Farmer Jones can either pay back the \$1000, or offer Farmer Smith the alternative of taking the horse instead. If Farmer Jones offers to pay back the loan, Farmer Smith takes his money and the game is over. If Farmer Jones offers the alternative, then Farmer Smith can either accept the offer or reject it. If he accepts the offer, then he gets the horse — and Farmer Jones does not have to pay him the \$1000. If he rejects the offer, then Farmer Jones has to pay him the \$1000 (but gets to keep the horse).

10. Find the expected revenue from an efficient auction, when there are three bidders for an object, and each bidder’s (true) willingness to pay for the object is an independent draw from the bivariate distribution : the willingness to pay is \$10 with probability 0.5 and \$20 with probability 0.5.