GS/ECON 5010 APPLIED MICROECONOMICS

Answers to Midterm Exam (of October 30 2017)

*Q*1. What are the Marshallian demand functions for a person whose preferences can be represented by the utility function

$$U(x_1, x_2, x_3) = x_1 + \log(x_2) + \log(x_3) \qquad ?$$

*A*1. The first–order conditions for the consumer's utility maximization, subject to her budget constraint, are

$$\frac{\partial U}{\partial x_1} = 1 = \lambda p_1 \tag{1-1}$$

$$\frac{\partial U}{\partial x_2} = \frac{1}{x_2} = \lambda p_2 \tag{1-2}$$

$$\frac{\partial U}{\partial x_3} = \frac{1}{x_3} = \lambda p_3 \tag{1-3}$$

where λ is the Lagrange multiplier associated with the budget constraint $p_1x_1 + p_2x_2 + p_3x_3 = y$.

Substituting from $(1-1)$ for λ into equations $(1-2)$ and $(1-3)$,

$$\frac{1}{x_2} = \frac{p_2}{p_1} \tag{1-4}$$

$$\frac{1}{x_3} = \frac{p_3}{p_1} \tag{1-5}$$

or

$$x_2^M(p_1, p_2, p_3, y) = \frac{p_1}{p_2} \tag{1-6}$$

$$x_3^M(p_1, p_2, p_3, y) = \frac{p_1}{p_3} \tag{1-7}$$

which are the Marshallian demand functions for goods 2 and 3.

From the person's budget constraint,

$$x_1(p_1, p_2, p_3, y) = \frac{1}{p_1}[y - p_2x_2^M(p_1, p_2, p_3, y) - p_3x_3^M(p_1, p_2, p_3, y)] \tag{1-8}$$

or (from equations $(1-6)$ and $(1-7)$)

$$x_1^M(p_1, p_2, p_3, m) = \frac{y}{p_1} - 2 \tag{1-9}$$

[Equation $(1-9)$ makes sense only if $y \geq 2p_1$. If the price of good 1 is high enough so that $y/p_1 < 2$, then the person will be at a corner solution. She will choose not to consume any of good 1, and will pick x_2 and x_3 so as to maximize $\log(x_2) + \log(x_3)$ subject to her budget constraint. That's maximizing a Cobb–Douglas utility function, so that in this case (the case in which $y < 2p_1$), her Marshallian demand functions will be

$$x_1^M(p_1, p_2, p_3, y) = 0 \qquad (1-10)$$

$$x_2^M(p_1, p_2, p_3, y) = \frac{y}{2p_2} \qquad (1-11)$$

$$x_3^M(p_1, p_2, p_3, y) = \frac{y}{2p_3} \quad] \qquad (1-12)$$

*Q*2. Give three distinct definitions of what it means for person A to be always more risk averse than person B.

*A*2. The following are all equivalent statements, each stating that person A is more risk averse than person B

1) If person A is willing to undertake a risky gamble g, then so is person B.

2) If person A's utility–of–wealth function is $u^A(W)$ and person B's utility–of–wealth function is $U^B(W)$, then

$$u^A(W) = F[U^B(W)]$$

where $F(\cdot)$ is an increasing, concave function.

3) For any risky gamble g, person A's certainty equivalent to the gamble is less than person B's.

4) For any risky gamble g, person A's risk premium for the gamble is greater than person B's.

5) For any level of wealth W, person A's coefficient of absolute risk aversion, $-u_A^{''}(W)/u_A^{'}(W)$ is greater than person B's $-U_B^{''}(W)/U_B^{'}(W)$.

6) For any level of wealth W, person A's coefficient of relative risk aversion, $-u_A^{''}(W)W/u_A^{'}(W)$ is greater than person B's $-U_B^{''}(W)W/U_B^{'}(W)$.

*Q*3. What is the **profit function** $\pi(p, w_1, w_2)$ for a perfectly competitive firm with a production function

$$f(x_1, x_2) = 3(x_1 x_2)^{1/3} \qquad ?$$

There are two paths to get to the same answer : (i) find directly the input combination (x_1, x_2) which maximizes profits $pf(x_1, x_2) - w_1x_1 - w_2x_2$, or (ii) first find the firm's cost function $C(w_1, w_2, y)$, and then find the output level y which maximizes the firm's profit $py - C(w_1, w_2, y)$.

(i) The "direct" method : maximization of $pf(x_1, x_2) - w_1 x_1 - w_2 x_2$ with respect to x_1 and x_2 yields first–order conditions

$$p x_1^{-2/3} x_2^{1/3} - w_1 = 0 \tag{3 – 1}$$

$$p x_1^{1/3} x_2^{-2/3} - w_2 = 0 \tag{3 – 2}$$

when $f(x_1, x_2) = (x_1 x_2)^{1/3}$.

Equations $(5-1)$ and $(5-2)$ imply that, when the firm maximizes profits,

$$x_2 = \frac{w_1}{w_2} x_1 \tag{3 – 3}$$

Substituting $(3-3)$ into $(3-1)$,

$$p[w_1]^{1/3}[w_2]^{-1/3} x_1^{-1/3} = w_1 \tag{3 – 4}$$

or

$$x_1(w_1, w_2, p) = p^3 [w_1]^{-2} [w_2]^{-1} \tag{3 – 5}$$

which is the unconditional input demand function for input (1). From $(3-3)$, the unconditional input demand function for input 2 is

$$x_2(w_1, w_2, p) = p^3 [w_1]^{-1} [w_2]^{-2} \tag{3 – 6}$$

Given these unconditional input demand functions, the firm's profit function is

$$\pi(w_1, w_2, y) = pf(x_1(w_1, w_2, p), x_2(w_1, w_2, p)) - w_1 x_1(w_1, w_2, p) - w_2 x_2(w_1, w_2, p) \tag{3 – 7}$$

Here

$$\begin{aligned} \pi(w_1, w_2, p) = {} & 3p[p^3[w_1]^{-2}[w_2]^{-1}]^{1/3}[p^3[w_1]^{-1}[w_2]^{-2}]^{1/3} \\ & - w_1[p^3[w_1]^{-2}[w_2]^{-1}] - w_2[p^3[w_1]^{-1}[w_2]^{-2}] \end{aligned} \tag{3 – 8}$$

which can be simplified to

$$\pi(w_1, w_2, p) = p^3 w_1^{-1} w_2^{-1} \tag{3 – 9}$$

(ii) The "indirect" method using the cost function.

The cost function comes from the minimization of $w_1 x_1 + w_2 x_2$ subject to the constraint $f(x_1, x_2) = y$. The first-order conditions for this minimization are

$$w_1 = \mu[x_1^{-2/3} x_2^{1/3}] \tag{3 – 10}$$

$$w_2 = \mu[x_1^{1/3} x_2^{-2/3}] \tag{3 – 11}$$

where μ is the Lagrange multiplier associated with the constraint $f(x_1, x_2) = y$.

Equations $(3-10)$ and $(3-11)$ imply that

$$x_2 = \frac{w_1}{w_2} x_1 \tag{3-12}$$

so that the constraint $f(x_1, x_2) = y$ can be written

$$3(w_1)^{1/3}(w_2)^{-1/3}(x_1)^{2/3} = y \tag{3-13}$$

which can be re–arranged into the conditional input demand function for x_1,

$$x_1(w_1, w_2, y) = [3^{-3/2}](w_1)^{-1/2}(w_2)^{1/2}y^{3/2} \tag{3-14}$$

From $(3-12)$, the conditional input demand function for x_2 is

$$x_2(w_1, w_2, y) = [3^{-3/2}](w_1)^{1/2}(w_2)^{-1/2}y^{3/2} \tag{3-15}$$

Which means that the cost function for the firm is $w_1x_1(w_1, w_2, y) + w_2x_2(w_1, w_2, y)$ or

$$C(w_1, w_2, y) = 2[3^{-3/2}](w_1)^{1/2}(w_2)^{1/2}y^{3/2} \tag{3-16}$$

The firm maximizes profits by choosing an output level y to maximize

$$py - C(w_1, w_2, y)$$

From $(3-16)$, the first–order condition for this maximization is

$$p - 3[3^{-3/2}](w_1)^{1/2}(w_2)^{1/2}y^{1/2} = 0 \tag{3-17}$$

or

$$y(w_1, w_2, p) = 3[(w_1)^{-1}(w_2)^{-1}p^2] \tag{3-18}$$

which is the firm's supply function. The firm's profit is $py(w_1, w_2, p) - C(w_1, w_2, y[w_1, w_2, p])$, so that

$$\pi(w_1, w_2, p) = 3[(w_1)^{-1}(w_2)^{-1}p^3] - 2[3^{-3/2}](w_1)^{1/2}(w_2)^{1/2}(3[(w_1)^{-1}(w_2)^{-1}p^2])^{3/2} \tag{3-19}$$

which can be simplified to

$$\pi(w_1, w_2, y) = (w_1)^{-1}(w_2)^{-1}p^3 \tag{3-20}$$

(which is the same as $(3-9)$).