Games of Incomplete Information

complication : a player might not know which node she is at

what?

example : player 1 moves first, but player 2 does not actually observe what action player 1 takes

so he doesn't know if he is at node L or R in the game, if L and R were player 1's initial actions

that's how a **simultaneous move** game can be represented in extensive form

it's just a sequential game in which player 2 does not observe player 1's move

Information Sets

an information set for a player is a collection of her decision nodes

an information set is where a player knows she is

so if player 1 moved first, choosing either L or R, and if player 2 did not observe player 1's choice, then **both** nodes, the one at the end of branch L and the one at the end of branch R are in the same information set

if player 2 has observed everything, then her information sets consist of single nodes

an information set with more than one node in it means that you have incomplete information

so extensive form game 4 is exactly the same game as strategic form game 9 :

player 2, when he moves, does not know whether player 1 has chosen t or b

if two nodes are in the same information set, then they must have exactly the same actions branching out of them

(why?)

revising the definition of a sub–game

before, I said that a sub–game to any game in extensive form was simply a (non–terminal) node and all the branches and nodes that followed from it

now I'll give the correct definition :

a sub–game is any (non–terminal) node **which is a "singleton" information set** — and all the branches and nodes that followed from it

extensive form game 5 has **no** sub–games (except for the game itself) ; after player 1 moves, player 2 knows either that (i) player 1 chose one of $\{1a, 1b, 1c\}$, or (ii) one of $\{1d, 1e\}$

Moves by "Nature"

so far, it has been assumed that the structure of each game is common knowledge

that is, player 1 knows player 2's payoffs (and player 2 knows that player 1 knows this, and player 1 knows that player 2 knows that she knows this, and so on...)

in lots of situations, that's not very plausible

Case 1: players are firms, actions are output levels, payoffs are profits

but firm 2 does not know exactly firm 1's cost function

and he needs to know this to calculate player 1's profits for any choice of outputs by the firms

Case 2 : players are firms bidding on rights to drill for oil in some area

each firm has done some testing in the area, to try and determine the likely yield

but firm 1 doesn't have access to firm 2's test results (and vice versa)

Case 3 : players are dictators, choosing whether to fight a war with each other

but player 1 does not know player 2's payoffs for sure : she does not know whether he is a "rational" agent trying to bluff (and avoid war if possible), or a "crazy" agent who actually gets a positive payoff from confrontation, even if there is no other tangible gain these cases can all be represented as games of incomplete information, using a trick

known as the “Harsanyi transform”

we add a fictitious third player : “nature”

nature gets no payoffs, and does not behave strategically : but nature has actions

so, in Case 1, nature moves first, and chooses whether firm 1 is “low cost”, or “high cost”

then player 1 moves, and then player 2

but player 1 does observe nature’s move

(i.e. : she knows her own cost)

and player 2 does not

in Case 2, nature again moves first, and chooses a pair of pieces of information : one piece for each firm

for example, one move in this game by nature might be (g, B) : meaning firm 1's test oil well results came back "good", and firm 2's came back "bad"

firm 1's information sets would be : $\{(g, G), (g, B)\}$,and $\{(b, G), (b, B)\}$

she knows her results, but not firm 2's

and firm 2's information sets would be $\{(g, G), (b, G)\}$ and $\{(g, B), (b, B)\}$

in Case 3, nature moves first, and chooses whether player 1 is “rational” or “crazy”

player 1 observes this move : she knows her own mind

but player 2 does not

so now player 1 knows what the payoffs are to each sequence of actions, but player 2 is not sure :

if the sequence of actions f by player 1, and then F by player 2 leads to a violent war, player 1 does not know if the payoff pair will be $(-100, -100)$ [i.e. player 2 is “rational”]

or $(-100, 100)$: player 2 is “crazy”

nature (or “chance”) is assumed to play randomly

so the uninformed player (or players) assign **prior** probabilities to each of nature’s moves

for example, in Case 3, player 1 may feel that there is a very small (but positive) chance that player 2 is crazy

so she views nature as choosing “crazy” with some small probability p, and “rational” with a large probability $1 - \pi$

extensive form 6 is an example of a game in which nature (called “chance” in the figure) moves first, and in which nature’s action is observed by player 1 but not by player 2 (as in Case 1)

note that player 2 has prior probabilities on what happens by chance : a 25% chance of the top move by chance,and a 75% chance of the bottom

also : her player 2 does get to observe player 1’s actions

her information sets show that she observes whether player 1 chose a or b, but not whether chance chose the top or bottom part of the tree