An Exchange Economy

no production : just re–allocation of given aggregate quantities of goods

$\mathbf{e} = (e_1, e_2, \ldots, e_n)$: aggregate **endowment** vector : aggregate quantities of each good

I : number of people

an allocation $\mathbf{x}$ is a list of I consumption bundles $\mathbf{x^i}$, one for each person

an allocation $\mathbf{x}$ is **feasible** if

$$\sum_{i=1}^{I} \mathbf{x}^i \leq \mathbf{e} \qquad (1)$$

or

$$\sum_{i=1}^{I} x_j^i \leq e_j \quad j = 1, 2, \cdots, n \qquad (2)$$

Pareto

if $u^i(\cdot)$ is a utility function representing person i's preferences, then

an allocation $\mathbf{x}$ is **Pareto–preferred** to another allocation $\mathbf{y}$ if and only if

$$u^i(\mathbf{x}^i) \geq u^i(\mathbf{y}^i) \quad i = 1, 2, \cdots, I \tag{3}$$

and

$$u^h(\mathbf{x}^h) > u^h(\mathbf{y}^h) \quad \text{for} \quad \text{some} \quad h \tag{4}$$

an allocation $\mathbf{x}$ is **Pareto efficient** if there is **no** feasible allocation $\mathbf{y}$ which is Pareto–preferred to $\mathbf{x}$

i.e. : $\mathbf{x}$ is Pareto efficient if there is no way to make one person better off (than she is with $\mathbf{x}$) without making someone else worse off

Condition for Pareto Efficiency

an allocation $\mathbf{x}$, with each person's consumption vector $\mathbf{x}^i >> 0$, is Pareto efficient if and only if

$$\frac{u^i_j}{u^i_k} = \frac{u^h_j}{u^h_k} \tag{5}$$

for any pair of people i and h, and any pair of commodities j and k

(needed the x^i_j's to be strictly positive for this — otherwise there might be corner solutions — on the boundary of the Edgeworth box)

An Example

$$u^1(\mathbf{x}^1) \equiv \ln x_1^1 + \ln x_2^1$$
$$u^2(\mathbf{x}^2) \equiv \ln x_1^2 + 4\ln x_2^2$$

so that

$$MRS^1 = \frac{x_2^1}{x_1^1}$$
$$MRS^2 = \frac{x_2^2}{4x_1^2}$$

and the efficiency condition (5) is

$$\frac{x_2^1}{x_1^1} = \frac{x_2^2}{4x_1^2} \tag{6}$$

Since an allocation must have $x_j^1 + x_j^2 = e_j$ to be feasible, condition (6) can be written

$$\frac{x_2^1}{x_1^1} = \frac{e_2 - x_2^1}{4(e_1 - x_1^1)} \tag{7}$$

Equation (7) defines a one–dimensional curve in the Edgeworth box ; implicitly it defines x_2^1 as a function of x_1^1. Any solution to (7) with $0 \leq x_2^1 \leq e_2$ and $0 \leq x_1^1 \leq e_1$ will be a Pareto efficient allocation.

(7) can be written

$$x_2^1 = \frac{e_2 x_1^1}{4e_1 - 3x_1^1} \tag{8}$$

Differentiation of (8) shows that the curve slopes up, and that it goes through the corners of the box : $x_2^1 = 0$ when $x_1^1 = 0$, and $x_2^1 = e_2$ when $x_1^1 = e_1$