Excess Demands

in general equilibrium, income is not exogenous

income is the value of a person’s endowment

\[ y^i = p \cdot e^i \equiv \sum_{j=1}^{n} p_j e^i_j \]  \hspace{1cm} (1)

consumer \( i \)'s problem is to find a consumption bundle \( x^i \) to

maximize \( u^i(x^i) \) subject to \( p \cdot x^i \leq y^i \equiv p \cdot e^i \)  \hspace{1cm} (2)
solution to maximization problem (2) could be written

\[ x^{iM}(p, p \cdot e^i) \]

where \( x^{iM} \) is the vector of Marshallian demand functions

Cobb–Douglas example: if \( u^i(x_1^i, x_2^i) = (x_1^i)^a(x_2^i)^{1-a} \) then

person \( i \)'s quantity demanded of good 1 is

\[ x_1^i = \frac{ay}{p_1} = \frac{a[p_1e_1^i + p_2e_2^i]}{p_1} \]

(3)

or

\[ x_1^i = ae_1^i + \frac{a}{p_1}p_2e_2^i \]

(4)
excess demands

since people have endowments of goods, their consumption of good $j$ comes from their own endowment of good $j$, plus any purchases made of good $j$ on the market (at least, if $x_j^i > e_j^i$)

**excess demand** $z_j^i$ for good $j$ is defined as the net purchase of good $j$ on the market

$$z^i(p; e_i) \equiv x^i_M(p, p \cdot e^i) - e^i$$  \hspace{1cm} (5)

$z_j^i < 0$ for some goods $j$ : person $i$ has to sell some of her endowment of one good, if she wants to buy anything on the market

since $p \cdot x^i = y^i \equiv p \cdot e^i$, therefore

$$p \cdot (z^i(p; e^i)) = 0$$  \hspace{1cm} (6)
Walras’s Law

equation (6) holds for any price vector $p$; it is a consequence of the person’s consumption bundle being on her budget line

add (6) over all $I$ consumers

let

$$Z(p) \equiv \sum^i z^i(p; e^i)$$

(of course, $Z$ depends as well on people’s endowments)

then

$$p \cdot Z(p) = 0$$  \hspace{1cm} (7)

equation (7) is called Walras’s Law; it holds for any price vector $p$
Walrasian Equilibrium

equilibrium in market $j$ means that aggregate demand for good $j$ equals aggregate supply of good $j$

here aggregate demand is $\sum_i x^{i \mathcal{M}}_j (p, p \cdot e^i)$; aggregate supply is just $e_j \equiv \sum_i e^i_j$

so equilibrium in market $j$ means that

$$Z_j(p) = 0$$

(8)

a vector $p$ clears all markets if (and only if)

$$Z(p) = 0$$

(9)

any price vector for which equation (9) holds is called a **Walrasian equilibrium** price vector