Excess Demands

in general equilibrium, income is not exogenous

income is the value of a person's endowment

$$y^i = \mathbf{p} \cdot \mathbf{e}^i \equiv \sum_{j=1}^{n} p_j e_j^i \tag{1}$$

consumer i's problem is to find a consumption bundle $\mathbf{x}^i$ to

$$\text{maximize} \quad u^i(\mathbf{x}^i) \quad \text{subject} \quad \text{to} \quad \mathbf{p} \cdot \mathbf{x}^i \leq y^i \equiv \mathbf{p} \cdot \mathbf{e}^i \tag{2}$$

solution to maximization problem (2) could be written

$$\mathbf{x}^{iM}(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i)$$

where $\mathbf{x}^{iM}$ is the vector of Marshallian demand functions

Cobb–Douglas example : if $u^i(x_1^i, x_2^i) = (x_1^i)^a (x_2^i)^{1-a}$ then

person i's quantity demanded of good 1 is

$$x_1^i = \frac{ay}{p_1} = \frac{a[p_1 e_1^i + p_2 e_2^i]}{p_1} \quad (3)$$

or

$$x_1^i = a e_1^i + a \frac{p_2}{p_1} e_2^i \quad (4)$$

excess demands

since people have endowments of goods, their consumption of good j comes from their own endowment of good j, plus any purchases made of good j on the market (at least, if $x_j^i > e_j^i$)

excess demand z_j^i for good j is defined as the net purchase of good j on the market

$$\mathbf{z}^i(\mathbf{p};\mathbf{e^i}) \equiv \mathbf{x}^{iM}(\mathbf{p}, \mathbf{p}\cdot\mathbf{e}^i) - \mathbf{e}^i \tag{5}$$

$z_j^i < 0$ for some goods j : person i has to sell some of her endowment of one good, if she wants to buy anything on the market

since $\mathbf{p}\cdot\mathbf{x}^i = y^i \equiv \mathbf{p}\cdot\mathbf{e}^i$, therefore

$$\mathbf{p}\cdot(\mathbf{z}^i(\mathbf{p};\mathbf{e}^i)) = 0 \tag{6}$$

Walras's Law

equation (6) holds for any price vector $\mathbf{p}$; it is a consequence of the person's consumption bundle being on her budget line

add (6) over all I consumers

let

$$\mathbf{Z}(\mathbf{p}) \equiv \sum^{i} \mathbf{z^i}(\mathbf{p}; \mathbf{e}^i)$$

(of course, $\mathbf{Z}$ depends as well on people's endowments)

then

$$\mathbf{p} \cdot \mathbf{Z}(\mathbf{p}) = 0 \tag{7}$$

equation (7) is called **Walras's Law** ; it holds for **any** price vector $\mathbf{p}$

Walrasian Equilibrium

equilibrium in market j means that aggregate demand for good j equals aggregate supply of good j

here aggregate demand is $\sum^i x_j^{iM}(\mathbf{p}, \mathbf{p} \cdot \mathbf{e}^i)$; aggregate supply is just $e_j \equiv \sum_i e_j^i$

so equilibrium in market j means that

$$Z_j(\mathbf{p}) = 0 \tag{8}$$

a vector $\mathbf{p}$ clears all markets if (and only if)

$$\mathbf{Z}(\mathbf{p}) = 0 \tag{9}$$

any price vector for which equation (9) holds is called a **Walrasian equilibrium** price vector