Short–Run Supply : An Example

(new) notation

x : quantity of variable input

$\bar{k}$: quantity of fixed input

q : quantity of output

$$q = (x^{\rho} + \bar{k}^{\rho})^{\beta/\rho} \tag{1}$$

let $\rho = -1$

so

$$q = [\frac{1}{x} + \frac{1}{\bar{k}}]^{-\beta} \tag{2}$$

or

$$q = [\frac{x\bar{k}}{x + \bar{k}}]^{\beta} \tag{3}$$

bounded maximum output : as $x \to \infty$, $q \to \bar{k}^{\beta}$

to produce q, $(x\bar{k})^{\beta} = (x + \bar{k})^{\beta} q$, or

$$x = \frac{\bar{k} q^b}{\bar{k} - q^b} \qquad q < \bar{k}^{\beta} \tag{4}$$

where $b \equiv 1/\beta$

total cost :

$$TC = w \frac{\bar{k} q^b}{\bar{k} - q^b} + r\bar{k} \tag{5}$$

so that

$$MC = w(\bar{k} b q^{b-1}) \frac{\bar{k}}{(\bar{k} - q^b)^2} \tag{6}$$

$$AVC = w(\bar{k} q^{b-1}) \frac{1}{\bar{k} - q^b} \tag{7}$$

$MC(q) > AVC(q)$**?**

$$MC = \frac{\bar{k}b}{\bar{k} - q^b} AVC \qquad (8)$$

$b \geq 1$ implies that $MC(q) > AVC(q)$ for all $q \geq 0$

but if $b < 1$, then $MC(q) > AVC(q)$ if and only if

$$q^b > (1 - b)\bar{k} \qquad (9)$$

$$MC'(q) = \frac{MC}{q}[b - 1 + \frac{bq^b}{\bar{k} - q^b}] \qquad (10)$$

which implies that $MC(q)$ curve is U–shaped if $b < 1$

$MC(q) > AVC(q)$ if

$$q > [(1 - b)\bar{k}]^{\beta} \qquad (11)$$

firm's short–run supply curve

invert the function

$$p = w(\bar{k}bq^{b-1})\frac{\bar{k}}{(\bar{k} - q^b)^2} \tag{12}$$

to get q as function of p