Collusion

q^i : output of firm i in the industry

J : number of firms in the industry

profit of firm j

$$\pi^j(q^1, q^2, \ldots, q^J)$$

with

$$\frac{\partial \pi^j}{\partial q^i} < 0 \quad i \neq j$$

special case : homogeneous products

$$Q \equiv q^1 + q^2 + \cdots + q^J$$

i.e. : perfect substitutes

so that

$$\pi^j = p(q^1 + q^2 + \cdots + q^J)q^j - C(q^j, \mathbf{w})$$

(back to the more general case)

joint profits of the industry

$$\Pi \equiv \sum_j \pi^j(q^1, q^2, \ldots, q^J) \qquad (1)$$

collusion : pick $(q^1, q^2, \ldots, q^J)$ so as to maximize Π

first–order conditions

$$\sum_j \frac{\partial \pi^j}{\partial q^i} = 0 \quad i = 1, 2, \cdots, J \qquad (2)$$

or

$$\frac{\partial \pi^i}{\partial q^i} = -\sum_{j \neq i} \frac{\partial \pi^j}{\partial q^i} \qquad i = 1, 2, \cdots, J \tag{3}$$

so, at this collusive optimum,

$$\frac{\partial \pi^i}{\partial q^i} > 0 \qquad i = 1, 2, \cdots, J$$

each firm has an incentive to deviate unilaterally from the collusive optimum

Incentives to Cooperate

if firms act unilaterally, they each choose q^i to maximize their own profits π^i

$$\frac{\partial \pi^i}{\partial q^i} = 0 \qquad i = 1, 2, \cdots, J \tag{4}$$

small decrease in q^i from this level has **no** effect on firm i's own profits, but increases the profits of each other firm

so if firm i and j each agree to decrease q^i and q^j slightly (from the levels firms set unilaterally),

$$\Delta \pi^i = \frac{\partial \pi^i}{\partial q^i} \Delta q^i + \frac{\partial \pi^i}{\partial q^j} \Delta q^j > 0 \tag{5}$$

$$\Delta \pi^j = \frac{\partial \pi^j}{\partial q^i} \Delta q^i + \frac{\partial \pi^j}{\partial q^j} \Delta q^j > 0 \tag{6}$$