Bertrand Duopoly

prices are the strategic variables

quantity sold by firm 1: \( q^1(p^1, p^2) \)

\[
\pi^1 = p^1q^1(p^1, p^2) - C[w, q^1(p^1, p^2)]
\] (1)

prices chosen simultaneously

(Nash) equilibrium: a pair of prices \((p^1, p^2)\), such that \(p^1\) maximizes \(\pi^1\), given \(p^2\), and such that \(p^2\) maximizes \(\pi^2\), given \(p^1\)
“benchmark” case

i homogeneous output; i.e. firm 1’s product is a perfect substitute for firm 2’s

ii constant returns to scale: \( C(w,q) \equiv cq \), where \( c \) is some constant (which depends on input prices)

market demand: \( D(p) \) is the equation of the market demand curve for the homogeneous product

homogeneous product \( \rightarrow \) buyers always buy from cheapest source

implication

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demand for firm $i$’s product

if $p^1 > p^2$, then $q^1(p^1, p^2) = 0$

why? everyone buys from (cheaper) firm #2

if $p^1 < p^2$, then $q^1(p^1, p^2) = D(p^1)$

everyone buys from firm #1

if $p^1 = p^2$, then

$$q^1(p^1, p^2) = q^2(p^1, p^2) = \frac{1}{2}D(p^1) \quad (2)$$

(rule (2) is not essential)
Nash equilibrium

\[ p^1 > p^2 > c \]

can’t be an equilibrium: firm #1 makes zero profits (since it has zero sales); given \( p^2 \), firm #1 can do better than that, by choosing some \( p' \) between \( c \) and \( p^2 \) (if \( c < p' < p^2 \), then firm #1 will get positive sales from charging the price \( p' \), and will make positive profits, since \( p' > c \))

similarly, \( p^2 > p^1 > c \) cannot be a Nash equilibrium

what about \( p^1 = p^2 > c \)?

can’t be an equilibrium
when $p^1 = p^2 > c$, firm 1’s profits are

$$\frac{1}{2} [p^2 - c] D(p^2)$$

by lowering its price very slightly, from $p^2$ to $p' = p^2 - \epsilon$, firm #1 lowers its profit margin very slightly, from $p^2 - c$ to $p' - c$

but this slight price reduction will more than **double** its sales: from $\frac{1}{2} D(p^2)$ to $D(p') > D(p^2)$

if $\epsilon$ is small enough ($p'$ close enough to $p^2$), this change in strategy must increase firm 1’s profits, so that $p^1 = p^2 > c$ cannot be a Nash equilibrium
what’s left?

how about \( p^1 > p^2 = c \)?

also can’t be a Nash equilibrium: firm #2 gets all the sales, but has zero profits (since its price equals its average cost); given \( p^1 \), firm #2 can increase profits by raising its price from \( p^2 = c \) to some \( p' \) with \( p^1 > p' > c \); if \( p' < p^1 \) firm #2 will still get all the sales, but if \( p' > c \) firm #2 will now make a positive profit per unit sold.

clearly there can be no Nash equilibrium in which **either** firm charged a price below cost: the lower–price firm will make negative profits; it always could do better by charging some price above \( c \), which guarantees profits are 0 or positive.
the unique Nash equilibrium in this market is 
\( p^1 = p^2 = c \)

if \( p^2 = c \), firm 1 makes zero profits by charging 
a price of \( p^1 = c \); but it cannot do better than that: 
any price above \( c \) gets it zero sales, and any price 
below \( c \) gives it negative profits

very different results than Cournot: with 
homogeneous output, and constant costs, a little 
competition is the same as perfect competition as 
long as the number of firms \( J \) in the market is 
greater than 1, then the equilibrium price will be \( c \), 
whether \( J \) is 2, or 3, or 1000