Gambles

$$g \equiv (p_1 \circ a_1, p_2 \circ a_2, \ldots, p_n \circ a_n)$$

means : g is a gamble, in which the outcome a_1 will arise with the probability p_1, the outcome a_2 will arise with the probability p_2, and so on

if, for example

$$g^1 \equiv (0.5 \circ 100, 0.5 \circ 0)$$

and

$$g \equiv (0.2 \circ 100, 0.8 \circ g^1)$$

then g is a compound gamble, for which : with 20 percent probability you get $100, and with 80 percent probability you get to play another gamble, in which you can win $100 with probability 50 percetnt and 0 with probability 50 percent.

Reducing Compound Gambles to Simple Gambles

$$g^1 \equiv (0.5 \circ 100, 0.5 \circ 0)$$

$$g \equiv (0.2 \circ 100, 0.8 \circ g^1)$$

$$g' \equiv (0.6 \circ 100, 0.4 \circ 0)$$

The gamble g' is a simple gamble which is **equivalent** to the compound gamble g.

von Neumann – Morgenstern Expected Utility

if a person's preferences over compound gambles obey axioms $G1$–$G6$ in *Jehle and Reny*, then the person's preferences can be represented by an **expected utility** function $u(\cdot)$

shorthand : let U stand for the person's ranking of **gambles**

if she prefers gamble g (weakly) to gamble g', then we could write $g \succeq g'$ — or $U(g) \geq U(g')$

[so U is defined over **gambles**, which are lists of payoffs a_i, and probabilities p_i with which the payoffs occur]

then the utility–of–wealth function $u(\cdot)$ is defined over values of wealth ; so lower–case u is just a function on levels of wealth

the utility–of–wealth function $u(\cdot)$ represents her preferences if for any two gambles

$$g \equiv (p_1 \circ a_1, p_2 \circ a_2, \ldots, p_n \circ a_n)$$
$$g' \equiv (p'_1 \circ a'_1, p'_2 \circ a'_2, \ldots, p'_N \circ a'_N)$$

then

$$g \succeq g'$$

if and only if

$$\sum_{i=1}^{n} p_i u(a_i) \geq \sum_{i=1}^{N} p'_i u(a'_i)$$

so, if the von Neumann–Morgenstern result applies, then, if

$$g \equiv (p_1 \circ a_1, p_2 \circ a_2, \ldots, p_n \circ a_n)$$

then

$$U(g) = \sum_{i=1}^{n} p_i u(a_i)$$

but here, monotonic transformations change things

if $u(a) = a$ and if

$$g = (1 \circ 5)$$

$$g' = (0.5 \circ 1, 0.5 \circ 9)$$

then

$$U(g) = u(5) = 5$$

and

$$U(g') = 0.5(1) + (0.5)9 = 5$$

so that $U(g) = U(g')$

but if $\tilde{u}(a) = \sqrt{(u(a))} = \sqrt{a}$,

$$U(g') = 0.5\sqrt{1} + 0.5\sqrt{9} = 2 < U(g) = \sqrt{5} \approx 2.24$$

Expected Value (and Expected Utility)

if

$$g = (p_1 \circ a_1, p_2 \circ a_2, \ldots, p_n \circ a_n)$$

then the **expected value** of any simple gamble g is

$$E(g) \equiv p_1 a_1 + p_2 a_2 + \cdots p_n a_n$$

risk–neutral : $u(E(g)) = U(g)$ for any simple gamble g

risk averse : $u(E(g)) > U(g)$ whenever g has any uncertainty

a risk lover : $u(E(g)) < U(g)$ whenever g has any uncertainty

Risk Aversion

the following statements are all equivalent :

i the person is risk averse

ii for this person, for a gamble g which has some uncertainty $g' \succeq g$, where g' is the outcome Eg with certainty

iii for this person, for a gamble g which has some uncertainty $u(Eg) > U(g)$, if $U(g)$ is her overall expected utility from some gamble, and $u(\cdot)$ is her von Neumann–Morgenstern utility–of–wealth function

iv for this person $u''(x) < 0$, if $u(\cdot)$ is her von Neumann–Morgenstern utility–of–wealth function

v if you offer this person a fair bet, that is a bet which pays her b_i with probability π_i, where $\sum_i \pi_i b_i = 0$, then she will reject the bet