## Certainty Equivalence

$$
\begin{gather*}
g=\left(p_{1} \circ W_{g},\left(1-p_{1}\right) \circ W_{b}\right) \quad W_{b}<W_{g} \\
U(g)=p_{1} u\left(W_{g}\right)+\left(1-p_{1}\right) u\left(W_{b}\right) \tag{1}
\end{gather*}
$$

the certainty equivalent $C E$ to $g$ is defined by

$$
\begin{equation*}
u(C E)=U(g)=p_{1} u\left(W_{g}\right)+\left(1-p_{1}\right) u\left(W_{b}\right) \tag{2}
\end{equation*}
$$

$u^{\prime \prime}(W)<0 \rightarrow C E<E(g)=p_{1} W_{g}+\left(1-p_{1}\right) W_{b}$
risk premium $P$ defined by

$$
\begin{gathered}
P \equiv E(g)-C E \\
u^{\prime \prime}(W)<0 \rightarrow P>0
\end{gathered}
$$

## Examples

$$
\begin{array}{rrr}
W_{b}=64 \quad W_{g} & =196 \quad p_{1}=0.5 \\
\text { so } E g=(0.5)(196+64) & =130 &
\end{array}
$$

$$
\begin{equation*}
u(W)=\sqrt{W} \tag{4}
\end{equation*}
$$

$\sqrt{( } C E)=(0.5) \sqrt{(196)}+(0.5) \sqrt{(64)}=11$
implying that $\sqrt{ }(C E)=11$ meaning that $C E=$ 121, and $P=9$

## but if

$$
\begin{equation*}
u(W)=\ln (\sqrt{W}) \tag{6}
\end{equation*}
$$

then

$$
\begin{equation*}
\ln (\sqrt{C E})=(0.5) \ln (\sqrt{196})+(0.5) \ln (\sqrt{64}) \tag{7}
\end{equation*}
$$

## or

$$
\begin{equation*}
\ln (\sqrt{C E})=(0.5) \ln 14+(0.5) \ln (8)=2.35925 \tag{8}
\end{equation*}
$$

implying that $\sqrt{(C E)}=e^{2.35925}$

$$
\text { or } C E=112, P=18
$$

# Arrow's and Pratt's 2 Measures 

$$
\begin{gather*}
R_{a}(W) \equiv-\frac{u^{\prime \prime}(W)}{u^{\prime}(W)}  \tag{9}\\
R_{r}(W) \equiv-\frac{u^{\prime \prime}(W) W}{u^{\prime}(W)}  \tag{10}\\
u(W)=\sqrt{W}
\end{gather*}
$$

if
then

$$
\begin{aligned}
& u^{\prime}(W)=\frac{1}{2 \sqrt{W}} \\
& u^{\prime \prime}=-\frac{1}{4} W^{-3 / 2}
\end{aligned}
$$

so that

$$
R_{a}(W)=\frac{1}{2} \frac{1}{W}
$$

if

$$
u(W)=\ln (\sqrt{W})
$$

then

$$
\begin{gathered}
u^{\prime}(W)=\frac{1}{\sqrt{W}} \frac{1}{2 \sqrt{W}}=\frac{1}{2 W} \\
u^{\prime \prime}=-\frac{1}{2 W^{2}}
\end{gathered}
$$

and

$$
R_{a}(W)=\frac{1}{W}
$$

