Certainty Equivalence

$$g = (p_1 \circ W_g, (1 - p_1) \circ W_b) \qquad W_b < W_g$$

$$U(g) = p_1 u(W_g) + (1 - p_1) u(W_b) \tag{1}$$

the certainty equivalent CE to g is defined by

$$u(CE) = U(g) = p_1 u(W_g) + (1 - p_1) u(W_b) \tag{2}$$

$$u''(W) < 0 \rightarrow CE < E(g) = p_1 W_g + (1 - p_1) W_b$$

risk premium P defined by

$$P \equiv E(g) - CE \tag{3}$$

$$u''(W) < 0 \rightarrow P > 0$$

Examples

$$W_b = 64 \quad W_g = 196 \quad p_1 = 0.5$$

so $Eg = (0.5)(196 + 64) = 130$

$$u(W) = \sqrt{W} \tag{4}$$

$$\sqrt{(}CE) = (0.5)\sqrt{(}196) + (0.5)\sqrt{(}64) = 11 \tag{5}$$

implying that $\sqrt{(}CE) = 11$ meaning that $CE = 121$,and $P = 9$

but if

$$u(W) = \ln(\sqrt{W}) \tag{6}$$

then

$$\ln(\sqrt{CE}) = (0.5)\ln(\sqrt{196}) + (0.5)\ln(\sqrt{64}) \tag{7}$$

or

$$ln(\sqrt{CE}) = (0.5)\ln 14 + (0.5)\ln(8) = 2.35925 \tag{8}$$

implying that $\sqrt{(CE)} = e^{2.35925}$

or $CE = 112, P = 18$

Arrow's and Pratt's 2 Measures

$$R_a(W) \equiv -\frac{u''(W)}{u'(W)} \tag{9}$$

$$R_r(W) \equiv -\frac{u''(W)W}{u'(W)} \tag{10}$$

if

$$u(W) = \sqrt{W}$$

then

$$u'(W) = \frac{1}{2\sqrt{W}}$$

$$u'' = -\frac{1}{4}W^{-3/2}$$

so that

$$R_a(W) = \frac{1}{2}\frac{1}{W}$$

if

$$u(W) = \ln(\sqrt{W})$$

then

$$u'(W) = \frac{1}{\sqrt{W}}\frac{1}{2\sqrt{W}} = \frac{1}{2W}$$

$$u'' = -\frac{1}{2W^2}$$

and

$$R_a(W) = \frac{1}{W}$$