GS/ECON 5010   Assignment 4   F2010

due : Wednesday December 1   8:30 am

Do all 5 questions. Each counts 20%.

1. Suppose that the aggregate demand curve by men for some good has the equation

$$Q^M = 150 - p$$

where Q^M is the aggregate quantity demanded by men, and p the price they pay. The aggregate demand curve of women, for the same product, is

$$Q^W = 300 - 5p$$

where Q^W is the aggregate quantity demanded by women.

A monopoly is able to supply the good at zero cost (in unlimited quantities). Compare the price paid by men, the price paid by women, and the monopoly's profit in the following two situations :

(i) The monopoly can charge different prices to men and women (who are not able to resell the good).

(ii) The monopoly must charge the same price to all buyers.

2. What is the equilibrium if Cournot duopolists, producing a homogeneous good, face an inverse demand curve

$$p = 10 - Q$$

(where Q is the aggregate quantity produced by the two firms, and p the resulting market price), if each firm's total cost of production is

$$TC(q) = q^2 + F \qquad \text{if} \qquad q > 0$$

(and $TC(0) = 0$), where $F > 0$ is each firm's positive fixed cost?

continued

3. What does the contract curve look like for a 2–person, 2–good exchange economy, with a total endowment of 5 units of good 1 and 20 units of good 2, if the preferences of the two people could be represented by the utility functions

$$u^1(x_1^1, x_2^1) = x_1^1 + 10x_2^1 - \frac{1}{2}[x_2^1]^2$$

$$u^2(x_1^2, x_2^2) = 10x_1^2 + x_2^2 - \frac{1}{2}[x_1^2]^2$$

where x_j^i is person i's consumption of good j?

4. What are the allocations of in the core of the following 3–person, 2–good economy? Person i's preferences can be represented by the utility function $u^i(x_1^i, x_2^i)$, where

$$u^1(x_1^1, x_2^1) = x_1^1 + x_2^1$$

$$u^2(x_1^2, x_2^2) = x_1^2 x_2^2$$

$$u^3(x_1^3, x_2^3) = x_1^3 + x_2^3$$

and the endowment vectors of the three people are $\mathbf{e}^1 = (1,1), \mathbf{e}^2 = (2,0), \mathbf{e}^3 = (0,2)$.

5. What is the competitive equilibrium in which there are 1 million people of type 1, and 1 million people of type 2, in which each type–1 person has an endowment vector $\mathbf{e}^1 = (7,1)$ and preferences represented by the utility function

$$u^1(x_1^1, x_2^1) = [x_1^1]^2 x_2^1$$

and each type–2 person has an endowment vector $\mathbf{e}^2 = (5,1)$ and preferences represented by the utility function

$$u^2(x_1^2, x_2^2) = x_1^2 + \ln(x_2^2) \qquad ?$$