GS/ECON 5010 Section "B" Assignment 3 F2013

due : Wednesday November 6 before class

Do all 5 questions. Each counts 20%.

1. What is the equation of the short–run supply curve of a firm which has a short–run total cost function with the equation

$$TC(q) = (q-12)^2 q + 48q + 100$$

where q is the quantity of output produced by the firm?

2. What is the long–run supply curve for the following competitive industry?

Firms differ in their cost of production t. The cost parameter t is distributed uniformly over some interval $[0, T]$.

The total cost of producing q units of output has the following form for a firm with cost parameter t :

$$TC = tq \qquad \text{if} \qquad q \leq 8$$

$$TC = tq + \frac{1}{2}(q-8)^2 \qquad \text{if} \qquad q > 8$$

3. A monopoly faces serves an equal number of two types of customers. The preferences of type–i customers ($i = 1, 2$) can be represented by the utility function

$$U^i(z, x) = z + a_i x - \frac{1}{2}x^2$$

where z is the person's consumption of a numéraire good, competitively supplied at a price of 1 per unit, x is the person's consumption of the good supplied by the monopoly, and a_i is a positive number, with $a_2 > a_1$. Each consumer has the same income M.

The monopoly can provide its good in individual "bundles" : a bundle j contains X_j units of the good, and has a total cost of P_j. Buyers cannot buy individual units of the monopoly's good ; they can't buy multiple bundles ; they either buy one bundle, or they buy nothing.

The monopoly's cost of production is c per unit of output.

What bundles should it offer, and what prices should it charge for each bundle?

4. What is the symmetric Bertrand equilibrium in a world in which all consumers have exogenous incomes y, all have CES preferences

$$U(\mathbf{x}) = ((x_1)^\rho + (x_2)^\rho + \cdots + (x_n)^\rho)^{1/\rho}$$

with $0 < \rho < 1$, in which each good is produced by a single producer with a constant marginal production cost c?

5. Suppose that a firm in a Cournot duopoly can raise its rival's marginal cost of production by some investment. This investment is costly, and has no effect on the firm's own marginal cost of production.

In particular, the total costs incurred by firm 1, if it produces q_1 units of output, are $c_1 q_1 + \beta(c_2)^2$ — and firm 2's costs are $c_2 q_2 + \beta(c_1)^2$, where β is a positive constant (with $\beta > 1/3$).

The decisions proceed as follows. First each firm chooses how much to invest in increasing the other firm's costs. That is, initially firm #1 chooses c_2, and incurs a cost of $\beta(c_2)^2$ and firm 2 chooses c_1 and incurs a cost of $\beta(c_1)^2$.

Each firm then observes what its own costs are, and the firms play a Cournot game (choosing output levels, given the marginal costs c_1 and c_2 which have already been determined). Aggregate demand for the firms' homogeneous output obeys the equation $p = a - Q$ for some positive a, where p is the market price, and Q the aggregate output in the industry.

(i) In a symmetric equilibrium, what cost levels c_2 and c_1 are chosen by the firms in the initial stage of the game (when they choose each others' costs, simultaneously and non–cooperatively)?

(ii) How do firms' equilibrium profits vary with the cost β of investment in cost–raising?