

due : Wednesday November 11    2:30 pm

Do all 5 questions. Each counts 20%.

1. Is the production function

$$f(\mathbf{x}) = 1 - \frac{1}{(x_1 + 1)(x_2 + 1)(x_3 + 1)}$$

weakly separable? Strongly separable? Explain briefly.

2. Derive the cost function for the production function

$$f(\mathbf{x}) = 1 - \frac{1}{(x_1 + 1)(x_2 + 1)(x_3 + 1)}$$

3. Derive the profit function for the production function

$$f(\mathbf{x}) = 1 - \frac{1}{(x_1 + 1)(x_2 + 1)(x_3 + 1)}$$

4. What is the long-run industry supply curve in an industry in which there are  $t$  firms of type  $t$ , where  $t = 1, 2, 3, \dots, 1000$ , in which the total cost function of a type- $t$  firm is

$$TC^t(y) = t \frac{y^2}{2} + y$$

where  $y$  is the firm's output?

5. What membership fee  $F$ , and what unit price  $p$ , should a monopoly charge in the following market?

There are equal numbers of two types (indexed by  $t$ ) of consumer. The monopoly must charge the same membership fee to all customers, and must charge the same unit price to all customers. The monopoly produces its product at zero cost (and so tries to maximize its total revenue). Each customer gets a utility of

$$U = u_0$$

if she buys nothing from the monopoly, and

$$U = u_0 - F - px + tx - \frac{1}{2}x^2$$

if she pays a membership fee  $F$  and buys  $x$  units of the monopoly's product at a price of  $p$  each.

For half the people,  $t = 10$ , and for the other half  $t = 20$ .

To purchase the monopoly's product (at a price of  $p$  per unit), a customer must pay the membership fee  $F$ . [So that a customer's two options are to pay the fee  $F$ , and then choose how many units  $x$  of the monopoly's product to buy at a price  $p$ , or (ii) not to pay the membership fee, and not to buy anything from the monopoly.]