GS/ECON 5010 Section "B" Assignment 4 F2015

due : Wednesday November 25 before class

Do all 5 questions. Each counts 20%.

1. Another model of duopoly is that of **von Stackelberg**, in which firms choose output levels **sequentially**. That is, firm 1 chooses its output quantity **first**, and cannot change that quantity after it has made its choice. Next, firm 2 observes what quantity firm 1 has chosen, and then chooses its own output quantity. What quantities would the 2 firms choose, if they behaved in this manner, if the cost of production (for each firm) were 0, and if the aggregate demand for the firms' homogeneous product were

$$Q = 12 - p$$

(where p is the market price, and Q the aggregate quantity demanded)?

2. Solve for the equilibrium quantities in a **3**–firm Stackelberg model, with the demand and cost functions from question #1 above.

[That is, firm 1 commits first to its quantity q_1. Firm 2 observes q_1 , and then commits to its own quantity q_2. Finally firm 3 observes q_1 and q_2, and then chooses its profit–maximizing output quantity q_3.]

3. What does the contract curve look like for a 2–person, 2–good exchange economy, with a total endowment of A units of good 1 and B units of good 2, if the preferences of the two people could be represented by the utility functions

$$u^1(x_1^1, x_2^1) = 1 - \frac{1}{x_1^1} - \frac{1}{x_2^1}$$

$$u^2(x_1^2, x_2^2) = \log(x_1^2) + x_2^2$$

where x_j^i is person i's consumption of good j?

4. What are the allocations in the core of the following 3–person, 2–good economy? Person i's preferences can be represented by the utility function $u^i(x_1^i, x_2^i)$, where

$$u^1(x_1^1, x_2^1) = x_1^1$$

$$u^2(x_1^2, x_2^2) = x_1^2 x_2^2$$

$$u^3(x_1^3, x_2^3) = x_2^3$$

and the endowment vectors of the three people are $\mathbf{e}^1 = (0, 4), \mathbf{e}^2 = (4, 0), \mathbf{e}^3 = (2, 2)$.

5. Find a competitive equilibrium to a 2–good, 3–million–person economy, in which 1 million people have preferences and endowments like person 1 in the previous question (# 4), 1 million people have preferences and endowments like person 2 in the previous question, and 1 million people have preferences and endowments like person 3 in the previous question. [That is, find a competitive equilibrium to an economy which is the economy of question #4 cloned one million times.]