# “is at least as good as”

Suppose that $\mathbf{x}^{\mathbf{1}}$ and $\mathbf{x}^2$ are 2 consumption **bundles**. That is, each of $\mathbf{x}^1$ and $\mathbf{x}^2$ are lists of $n$ numbers, where $n$ is the number of different commodities that the consumer can consume. [Formally, $\mathbf{x}^1$ and $\mathbf{x}^2$ are each vectors in $\mathcal{R}^n_+$, the set of vectors with $n$ non–negative elements.]

The symbol $\succeq$ is simply a shorthand for “is at least as good as”. So

$$\mathbf{x}^1 \succeq \mathbf{x}^{\mathbf{2}} \tag{1}$$

is a shorthand for the expression :

“the person likes the consumption bundle $\mathbf{x}^1$ at least as much as the consumption bundle $\mathbf{x}^2$”.

The symbol $\succeq$ is **not** the same as the symbol $\geq$.

$$\mathbf{x}^1 \geq \mathbf{x}^2 \tag{2}$$

is a shorthand for the mathematical expression :

“every element in the vector $\mathbf{x}^1$ is at least as large as the corresponding element in the vector $\mathbf{x}^2$”.

That is, $\mathbf{x}^1 \geq \mathbf{x}^2$ means that $x_1^1 \geq x_1^2$, $x_2^1 \geq x_2^2$, $x_3^1 \geq x_3^2$, and so on, where $x_i^1$ is the $i$–th element in the vector $\mathbf{x}^1$.

So the expression $\succeq$ does not need to have any relation at all with the idea of “greater than or equal” ($\geq$) — **unless** specific assumptions [such as **strict monotonicity**] are made about the preferences.

For example : what if $A$ and $B$ are objects that cannot be assigned numerical values? $A$ and $B$ could be political candidates, or songs, or flavours of ice cream. People still have preferences over political candidates, and songs, and flavours of ice cream. So we could write the shorthand

$$A \succeq B$$

meaning that : “the person likes the political candidate $A$ at least as much as the political candidate $B$” (or “the person likes the song $A$ at least as much as the song $B$”), even though it would make **no sense** to write $A \geq B$, since it does not make much sense to say that “a political candidate is at least as big as another candidate”, or “a song is at least as big as another song”.

# Consumption Bundles

It **does** make sense to assign numerical values to consumption bundles. A consumption bundle $\mathbf{x}^1$ is a list of **quantities** of different goods and services. Mathematically, $\mathbf{x}^1 \geq \mathbf{x^2}$ means that consumption bundle 1 offers at least as much of **every** good and service as bundle 2. And $\mathbf{x}^1 >> \mathbf{x}^2$ means that consumption bundle 1 offers **strictly more** of every good and service than bundle 2.

If preferences are **strictly monotonic**, then the person wants more of each good and service, so that $\mathbf{x}^1 \succeq \mathbf{x}^2$ whenever $\mathbf{x}^1 \geq \mathbf{x}^2$ ; and $\mathbf{x}^1 \succ \mathbf{x}^2$ whenever $\mathbf{x}^1 >> \mathbf{x}^2$.

But the notion of $\succeq$ still is a different notion than the notion of $\geq$. Typically, with strictly monotonic preferences, there will be lots of consumption bundles $\mathbf{x}^3$ and $\mathbf{x}^4$ such that bundle #3 offers more of some goods, and bundle #4 offers more of other goods, and yet the consumer can rank them, perhaps feeling that bundle #3 is better. So here it would **not** be true that $\mathbf{x}^3 \geq \mathbf{x}^4$, or that $\mathbf{x}^4 \geq \mathbf{x}^3$, but it would be true that $\mathbf{x}^3 \succeq \mathbf{x}^4$.