Shepherd's Lemma

$$e(\mathbf{p}, u) = \sum_{j=1}^{n} p_j x_j^h(\mathbf{p}, u) \tag{1}$$

differentiate (1) with respect to p_i,

$$\frac{\partial e(\mathbf{p}, u)}{\partial p_i} = x_i^h(\mathbf{p}, u) + \sum_{j=1}^{n} p_j \frac{\partial x_j^h}{\partial p_i} \tag{2}$$

must prove : second term on right side of (2) is zero

since utility is held constant, the change in the person's utility

$$\Delta u \equiv \sum_{j=1}^{n} \frac{\partial u}{\partial x_j} \frac{\partial x_j^h}{\partial p_i} = 0 \tag{3}$$

when p_i is changed, but the person is compensated so as to stay on the same indifference curve

from the first–order conditions for expenditure minimization

$$p_j = \mu \frac{\partial u}{\partial x_j} \quad j = 1, 2, \ldots, n \tag{4}$$

where μ is the Lagrange multiplier

so

$$\sum_j p_j \frac{\partial x_j}{\partial p_i} = \mu [\sum_j \frac{\partial u}{\partial x_j} \frac{\partial x_j^h}{\partial p_i}] = 0 \tag{5}$$

Substituting (5) into (2) yields

$$\frac{\partial e(\mathbf{p}, u)}{\partial p_i} = x_i^h(\mathbf{p}, u) \tag{6}$$

which is Shepherd's Lemma.