Slutsky Equation

u : the level of utility that a person gets when she chooses her best consumption bundle, given prices $\mathbf{p}$ and income y

it then follows that

$$x_i^h(\mathbf{p}, u) = x_i^m(\mathbf{p}, e(\mathbf{p}, u)) \tag{1}$$

and that's true whatever are the prices, as long as $u = v(\mathbf{p}, y)$.

So we can differentiate both sides of (1) with respect to any price p_j, and both sides of the equation must be equal. So

$$\frac{\partial x_i^h(\mathbf{p}, u)}{\partial p_j} = \frac{\partial x_i^m(\mathbf{p}, y)}{\partial p_j} + \frac{\partial x_i^m(\mathbf{p}, y)}{\partial y}\frac{\partial e(\mathbf{p}, y)}{\partial p_j} \tag{2}$$

where the differentiation comes using the chain rule, and recognizing that $y = e(\mathbf{p}, u)$. Now Shepherd's Lemma said that

$$\frac{\partial e(\mathbf{p}, u)}{\partial p_j} = x_j^h(\mathbf{p}, u) \tag{3}$$

and equation (1) says that

$$x_j^h(\mathbf{p}, u) = x_j^m(\mathbf{p}, y) \tag{4}$$

which means that (2) can be written

$$\frac{\partial x_i^h(\mathbf{p}, u)}{\partial p_j} = \frac{\partial x_i^m(\mathbf{p}, y)}{\partial p_j} + \frac{\partial x_i^m(\mathbf{p}, y)}{\partial y} x_j^m \tag{5}$$