

time=2.5 hours

Do any **6** of the following 10 questions. All count equally.

1. If a person's preferences can be represented by the utility function

$$u(x_1, x_2) = 100 - \frac{1}{x_1} - \frac{1}{x_2}$$

find the person's Marshallian demand functions for each good, her indirect utility function, her Hicksian demand functions, and her expenditure function.

2. Suppose that a person's utility-of-wealth function were

$$U(W) = 100 - e^{-\alpha W}$$

for $\alpha > 0$.

If there was a probability π that she would lose an amount L of her wealth, due to a fire, how much fire insurance would she buy, if she could buy I dollars worth of fire insurance for pI , where $p \geq \pi$?

3. State and prove the relation between a firm's (total) cost function, and its conditional input demand functions [Shephard's Lemma].

continued

4. What would be the equilibrium price and quantity in the long run, in a competitive industry in which there were many identical firms, each with the same long run total cost function

$$TC(q) = q^3 - 12q^2 + 58q$$

where q was the output of the firm, if the market demand curve for the output of the firms had the equation

$$Q = \frac{6600}{p}$$

where Q was the total quantity demanded, and p the price of the good?

5. In a Cournot duopoly, if total market demand for the firms' homogeneous product was

$$Q^D = A - p$$

where p is the market price, and if firm i 's total cost of producing q units was

$$TC_i(q) = c_i q$$

where c_1 and c_2 were positive constants, how would firm 1's equilibrium output and profits vary with its rival's production cost c_2 ?

6. Show that every competitive equilibrium allocation in an exchange economy must be Pareto optimal.

continued

7. What are all the allocations in the core of a 3–person, 2–good economy, in which each person’s preferences can be represented by the utility function

$$u^i(x_1^i, x_2^i) = x_1^i x_2^i$$

where x_j^i is person i ’s consumption of good j , and where the endowments e^i of the three people are $e^1 = (2, 0)$, $e^2 = (0, 2)$, $e^3 = (1, 1)$?

8. Find all the Nash equilibria (in pure or mixed strategies) to the game depicted below in strategic form.

	<i>L</i>	<i>R</i>
<i>t</i>	(10, 8)	(2, 2)
<i>m</i>	(8, 10)	(0, 6)
<i>b</i>	(2, 2)	(4, 6)

continued

9. Find a subgame perfect Nash equilibrium to the following game :

Firm 1 moves first, choosing whether or not to enter a market. If it chooses to enter, it must pay a entry fee of 20.

Firm 2 moves second, after observing firm 1's move. It chooses whether to enter or not, also paying an entry fee of 20 if it chooses to enter.

In the third stage, the firms play a Cournot game if they have both chosen to enter. If only one firm chose to enter, it acts as a single-price monopoly. If neither firm chose to enter, they do nothing in the third stage.

Each firm's marginal cost of production is **zero**, if it has chosen to enter (and paid its entry fee). Output of each firm is identical, and the market demand for the output has the equation

$$Q = 12 - p$$

where Q is the total quantity demanded, and p the price.

10. Suppose that there are four bidders in a second-price sealed-bid auction. Each bidder's value of the object being auctioned off is an independent draw from the same distribution. Each bidder values the object at \$96 with probability 0.5, and at \$48 with probability 0.5.

What is the expected revenue from this auction?