

time=2.5 hours

Do any **6** of the following 10 questions. All count equally.

1. If a person's preferences can be represented by the utility function

$$u(x_1, x_2, x_3) = x_1 + \ln x_2 + \ln(x_2 + x_3)$$

find the person's Marshallian demand functions for each good, her indirect utility function, her Hicksian demand functions, and her expenditure function, when $p_2 > 2p_3$.

2. A person has a fixed amount of wealth W , which she must allocate between an asset with a certain return of $1 + r_0$, and a risky asset, for which the return will be $1 + r_b$ with probability $1 - \pi$ and $1 + r_g$ with probability π (with $r_g > r_b$, and with $\pi r_g + (1 - \pi)r_b > r_0$).

How much will she invest in the risky asset, if she is a risk-averse expected utility maximizer with utility-of-wealth function $U(W) = \ln W$?

3. Show why the conditional demand for an input cannot be an increasing function of the price of that input.

continued

4. Is it possible that the long-run supply curve for a competitive industry had a negative slope? Explain.

5. What are the Cournot-Nash equilibria to a duopoly in which firms choose simultaneously the quantities to produce of a homogeneous good, if the demand function for the good has the equation

$$Q = 60 - p$$

and each firm's total cost function has the equation

$$\begin{aligned} TC(q) &= 120 + 30q & q > 0 \\ &= 0 & q = 0 \end{aligned}$$

where Q is total industry output, and p is the price?

6. An exchange economy consists of 4 people, each of whom has the same preferences, which can be represented by the utility function

$$u(x_1^h, x_2^h) = x_1^h x_2^h$$

(where x_j^h is person h 's consumption of good j).

Person 1 and 2 each have an endowment of 2 units of good 1 and none of good 2. Person 3 and 4 each have an endowment of 2 units of good 2 and none of good 1.

Give an example of an allocation which is Pareto optimal, which each person prefers to her initial endowment, but which is **not** in the core of the exchange economy.

continued

7. Calculate a competitive (“Walrasian”) equilibrium to the 2–person, 2–good exchange economy in which person 1’s preferences can be represented by the utility function

$$u^1(x_1^1, x_2^1) = x_1^1 + 5 \ln x_2^1$$

and person 2’s preferences by the utility function

$$u^2(x_1^2, x_2^2) = x_1^2 x_2^2$$

if person 1’s endowment of the two goods is $\mathbf{e}^1 = (4, 1)$ and person 2’s is $\mathbf{e}^2 = (2, 2)$.

8. Find all the Nash equilibria (in pure or mixed strategies) to the game depicted below in strategic form.

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
I	(3, 12)	(5, 0)	(5, 8)	(1, 4)
II	(4, 4)	(5, 3)	(10, 6)	(3, 10)
III	(1, 8)	(2, 8)	(7, 5)	(0, 4)
IV	(6, 4)	(2, 2)	(8, 5)	(4, 6)

9. Suppose that there are two bidders in an “English” (ascending–bid) auction. Each bidder’s value of the object being auctioned off is an independent draw from the same distribution. Each bidder values the object at \$5 with probability 0.5, and at \$10 with probability 0.5.

Show that the auctioneer can increase her expected revenue from this auction by introducing a **reserve price** : a price r which the winning bid must exceed (so that the object does not get sold if no bid is as high as this reserve price).

continued

10. Draw the extensive form diagram for the following game of asymmetric equilibrium, and find a perfect Bayesian equilibrium to it.

Player 1 is a candidate for a civil service job, and player 2 is the manager making the hiring decision.

Player 1 is either a good civil servant, or a bad civil servant. Player 1 knows whether she is a good or bad civil servant, but player 2 cannot observe this directly. Player 2's prior belief is that player 1 is a bad civil servant with probability $3/4$ (and good with probability $1/4$).

Player 1 moves first, choosing whether or not to get an MA degree.

Player 2 observes whether player 1 got a degree, and then makes his move, choosing whether or not to hire her.

Player 1 gets a payoff of 4 if she is hired, and 0 if she is not hired — regardless of whether she is a good or bad civil servant.

Player 2 gets a payoff of +2 from hiring a good civil servant, -2 from hiring a bad civil servant, and 0 if she chooses not to hire the candidate. Whether the candidate has an MA or not does not affect this payoff.

It is costly for player 1 to get an MA. (This payoff must be subtracted from her payoff of 4 or 0 from the hiring stage, if she chose to get an MA.) The cost of getting an MA is 1 for a good civil servant, but 5 for a bad civil servant.

[So, for example, if the manager hired a candidate with an MA, and that candidate turned out to be a good civil servant, then player 1 would have a payoff of $4 - 1 = 3$, and player 2 would have a payoff of 2. If the manager chose not to hire a candidate with an MA, and that candidate were a bad civil servant, then player 1 would get a payoff of -5 and player 2 would have a payoff of 0.]

end