GS/ECON 5010 answers to midterm October 2007

$Q1$. A consumer's expenditure function must be homogeneous of degree t in prices.
What is t?
Explain briefly.

$A1$. Here $t = 1$; the expenditure function is homogeneous of degree 1. To see this, consider the consumer's cost minimization problem (for which the expenditure function is the solution). Suppose that the quantities $x_1^*, x_2^*, \ldots, x_n^*$ solve this minimization. That means that

$$U(x_1^*, x_2^*, \ldots, x_n^*) = u \qquad (1-1)$$

$$\mathbf{p} \cdot \mathbf{x}^* \leq \mathbf{p} \cdot \mathbf{x} \quad \text{for} \quad \text{any} \quad \mathbf{x} \quad \text{with} \quad U(\mathbf{x}) \geq u \qquad (1-2)$$

where $\mathbf{p}$ is the vector of prices, and u is the required level of utility for the consumer.

Now suppose that all prices increase by a factor of k, to $(kp_1, kp_2, \cdots, kp_n)$. Equation $(1-1)$ still holds, and equation $(1-2)$ implies that

$$k\mathbf{p} \cdot \mathbf{x}^* \leq k\mathbf{p} \cdot \mathbf{x} \quad \text{for} \quad \text{any} \quad \mathbf{x} \quad \text{with} \quad U(\mathbf{x}) \geq u \qquad (1-3)$$

so that $\mathbf{x}^*$ is still the cost–minimizing consumption bundle when $k\mathbf{p}$ is the vector of prices. (This result shows that these cost–minimizing bundles, the Hicksian demands, are homogeneous of degree 0 in prices.)

Now

$$e(\mathbf{p}, u) = \sum_{i=1}^{n} p_i x_i^* \qquad (1-4)$$

so that

$$e(k\mathbf{p}, u) = \sum_{i=1}^{n} kp_i x_i^* = ke(\mathbf{p}, u) \qquad (1-5)$$

proving that the expenditure function is homogeneous of degree 1 in prices.

$Q2$. Derive the Hicksian (compensated) demand functions for a consumer whose preferences can be represented by the direct utility function

$$u(x_1, x_2) = 12 - \frac{1}{(x_1)^2} - \frac{1}{(x_2)^2}$$

$A2$. Solving the problem of minimizing $p_1x_1 + p_2x_2$ subject to $12 - \frac{1}{(x_1)^2} - \frac{1}{(x_2)^2} = u$ implies first–order conditions

$$\frac{2}{(x_1)^3} = \mu p_1 \qquad (2-1)$$

$$\frac{2}{(x_2)^2} = \mu p_2 \qquad (2-2)$$

where μ is the Lagrange multiplier on the utility constraint. Equations $(1-1)$ and $(1-2)$ imply that

$$x_2 = [\frac{p_1}{p_2}]^{1/3} x_1 \tag{2-3}$$

Substituting $(2-3)$ into the utility constraint,

$$12 - \frac{1}{(x_1)^2} - [\frac{p_2}{p_1}]^{2/3} \frac{1}{(x_1)^2} \tag{2-4}$$

or

$$(12-u)(x_1)^2 = \frac{(p_1)^{2/3} + (p_2)^{2/3}}{(p_1)^{2/3}} \tag{2-5}$$

Dividing both sides by $12-u$, and taking square roots,

$$x_1 = (12-u)^{-1/2}[(p_1)^{2/3} + (p_2)^{2/3}]^{1/2}(p_1)^{-1/3} \tag{2-6}$$

which is the Hicksian demand function for good 1. Similarly

$$x_2 = (12-u)^{-1/2}[(p_1)^{2/3} + (p_2)^{2/3}]^{1/2}(p_2)^{-1/3} \tag{2-7}$$

Since $e(\mathbf{p}, u) = p_1 x_1^h(\mathbf{p}, u) + p_2 x_2^h(\mathbf{p}, u)$, equations $(2-5)$ and $(2-6)$ imply that

$$e(\mathbf{p}, u) = (12-u)^{-1/2}[(p_1)^{2/3} + (p_2)^{2/3}]^{1/2}[(p_1)^{2/3} + (p_2)^{2/3}]$$

or

$$e(\mathbf{p}, u) = (12-u)^{-1/2}[(p_1)^{2/3} + (p_2)^{2/3}]^{3/2} \tag{2-8}$$

Differentiation of equation $(2-8)$ with respct to x_1 and x_2 respectively yields the expressions on the right side of equations $(2-6)$ and $(2-7)$, confirming Shephard's Lemma.

*Q*3. An expected–utility–maximizing person has utility of wealth function

$$U(W) = \frac{1}{1-\beta} W^{1-\beta} \quad \beta > 0$$

For what value of π will the person be willing to accept a gamble which doubles her wealth with probability π, and loses her all her wealth with probability $1-\pi$?

*A*3. If the person does not accept the gamble, her expected utility is

$$EU_{init} = \frac{1}{1-\beta} W^{1-\beta} \tag{3-1}$$

If she accepts the gamble, her expected utility is

$$EU_{gamble} = \pi \frac{1}{1-\beta} (2W)^{1-\beta} \tag{3-2}$$

Therefore, she will accept the gamble if and only if

$$\pi 2^{1-\beta} > 1 \qquad (3-3)$$

or

$$\pi > 2^{\beta-1} \qquad (3-4)$$

Her willingness to accept the gamble does not depend on her initial wealth (since her utility–of–wealth function displays a constant coefficient of relative risk aversion) ; the required probability of winning π is an increasing function of her coefficient β of relative risk aversion ; the required probability π must exceed 1/2.

And she will be unwilling to accept the bet, for any $\pi < 1$, if her coefficient of relative risk aversion is 1 or more.

$Q4$. Derive the long–run supply curve for an industry consisting of a large number of identical firms, each of which has a long–run average cost curve with the equation

$$AC(q) = q^2 - 12q + 50$$

where q is the firm's output.

$A4$. Here each firm has the same, U–shaped average cost curve.

Why is it U–shaped?

$$AC'(q) = 2q - 12$$

which is negative when $q < 6$ and positive when $q > 6$.

So each firm's average cost reaches a minimum at $q = 6$, at which point $AC = 14$.

At $q = 6$, $MC = AC = 14$, since MC must equal AC at the minimum of the average cost curve. (You can check this here. Here $TC = q^3 - 12q^2 + 50q$ so that $MC = 3q^2 - 24q + 50$.)

Each firm chooses an output level such that $p = MC$. Free entry and exit by firms in the long run implies that profits are zero for each identical firm, so that $p = AC$. If $p = AC = MC$, then each firm must be producing 6 units, at a marginal (and average) cost of 14.

So the long run supply curve for the industry is horizontal, at a height of 14.