

time=2.5 hours

Do any **6** of the following 10 questions. All count equally.

1. If a person's **indirect utility function** can be written

$$v(p_1, p_2, y) = \frac{y^2}{4p_2(p_1 - p_2)}$$

where p_1 and p_2 are prices of 2 goods, and y the consumer's income (and if $p_1 > 2p_2$), then

- (i) What is the equation of the consumer's expenditure function?
- (ii) What are the consumer's compensated demand functions for goods 1 and 2?
- (iii) Are goods 1 and 2 net complements or net substitutes?

2. If a person had a utility of wealth function

$$U(W) = W^a \quad ; \quad 0 < a < 1$$

and earned a salary Y , how much would she be willing to pay for full insurance against losing her job, if she had no other source of income other than her job, and if she expected the probability of losing her job was β ?

3. Prove that a firm's conditional demand for an input cannot be an increasing function of the price of that input.

continued

4. What is the equation of the supply curve of a firm which has a long-run total cost function with the equation

$$TC(q) = 6q^2 - 36q + 216\frac{q}{q+1}$$

where q is the quantity of output produced by the firm?

(You do not need to solve for the equation of the supply curve in closed form, just the relation between price and output supplied).

5. Show that one firm's profit must be an increasing function of its rival's marginal cost of production in a Cournot duopoly in which (i) the demand function for the two firms' homogeneous product is a straight line and (ii) each firm produces under constant returns to scale, where (iii) the firms' marginal costs may be different from each other's.

6. An exchange economy consists of 3 people, each of whom has the same preferences, which can be represented by the utility function

$$u(x_1^h, x_2^h) = x_1^h x_2^h$$

(where x_j^h is person h 's consumption of good j).

Person 1 and 2 each have an endowment of 2 units of good 1 and none of good 2. Person 3 has an endowment of 4 units of good 2 and none of good 1.

Give an example of an allocation which is Pareto optimal, which each person prefers to her initial endowment, but which is **not** in the core of the exchange economy.

continued

7. Prove the first fundamental theorem of welfare economics for an exchange economy, that every competitive equilibrium allocation is Pareto optimal.

8. Suppose 2 firms can produce a homogeneous product at zero cost. The aggregate demand for the product, as a function of its price, is

$$Q = 3 - p$$

The firms behave as Bertrand duopolists, setting prices simultaneously. (If they charge the identical price, they split the market evenly.)

The firms must set integer prices : $p \in \{0, 1, 2, 3\}$.

(i) Write down the strategic form of this game.

(ii) Is there a dominant strategy equilibrium? If not, can the game be solved by iterated elimination of weakly dominated strategies?

(iii) Find all the Nash equilibria to the game.

9. Find all the Nash equilibria (including those involving mixed strategies) to the following game in strategic form :

	<i>L</i>	<i>C</i>	<i>R</i>
<i>t</i>	(6, 4)	(0, 2)	(5, 2)
<i>m</i>	(4, 2)	(1, 5)	(4, 0)
<i>b</i>	(4, 6)	(2, 8)	(8, 3)

continued

10. What is the sub-game perfect Nash equilibrium to the following game?

Player A has a licence to sell beer, and player B owns a brewery. They want to sell beer to economists during a 3-day economists' convention. The 2 players must reach an agreement for sharing the profits ; with no agreement they cannot sell any beer. With an agreement, they can make \$1000 a day total profit from selling beer. They must decide how to split the profits between them.

The payoff to each player is simply her or his share of the profit from selling beer.

At the beginning of day 1, player A gets to propose to player B how they should split the profits. (So she proposes some share s , between 0 and 100%, which will be her share of the profits.) Player B can either accept or reject A's proposal. If he accepts, then they split the three days' profits (of \$3000) according to A's proposal. No further negotiation takes place.

If B rejects A's initial proposal, then they do not get to sell any beer on the first day. But (if he rejects A's initial proposal), B gets to make his own proposal at the beginning of the second day. Then A would get to accept or reject B's second-day proposal. If A accepts, they then split the 2 days' profits of \$2000 according to B's proposal.

If A rejects B's second-day proposal, then they cannot sell any beer on the second day. But [if the first two days' proposals have both been rejected], A gets to make a new proposal on how to split the profits, on the morning of the third day. If B agrees to this new proposal, they split the last day's profits according to the proposal. If B rejects this last offer, then the game is over, and they have no profits to split.

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