

**time=2.5 hours**

Do any **6** of the following 10 questions. All count equally.

1. If a person's preferences can be represented by the utility function

$$u(x_1, x_2, x_3) = x_1 + 100 - \frac{1}{x_2} + \ln x_3$$

find the person's Marshallian demand functions for each good, her indirect utility function, her Hicksian demand functions, and her expenditure function.

2. Person A is a risk-averse expected utility maximizer, with utility-of-wealth function  $u(W)$ . Person B is also an expected utility maximizer, with utility-of-wealth function  $V(W) = f[u(W)]$  where  $f(\cdot)$  is an increasing concave function, and where  $u(\cdot)$  is person A's utility-of-wealth function.

Show that person B has a higher risk premium for any risky gamble than person A.

3. What is a firm's cost function, if its production function is

$$y = (\sqrt{x_1} + \sqrt{x_2})^3$$

where  $y$  is the quantity of output, and  $x_1$  and  $x_2$  are the quantities used of two inputs?

**continued**

4. What is the equation of the long-run industry supply curve of a perfectly competitive industry in which there are a large number of identical firms, each of which has the same total cost function

$$TC(y) = 2y^3 - 48y^2 + 388y$$

where  $TC(y)$  is the total cost of producing  $y$  units of output?

5. How does the equilibrium price vary with the number of firms  $n$ , in the following  $n$ -firm model of oligopoly?

Firms produce an identical good. Each firm has the same constant-returns-to-scale technology, so that the total cost of producing  $y$  units of the good is  $cy$  for any firm. Firms choose their prices simultaneously and non-cooperatively, and buyers buy from the lowest-cost firm. (If 2 or more firms are tied with the lowest price, they split the market evenly.)

6. What are all the Pareto efficient allocations in the following 3-person exchange economy?

There are 2 goods : the aggregate endowment of good 1 is 30 units, and the aggregate endowment of good 2 is 40 units.

Person 1's preferences can be represented by the utility function  $U^1(x_1^1, x_2^1) = x_1^1 + x_2^1$ .

Person 2's preferences can be represented by the utility function  $U^2(x_1^2, x_2^2) = x_1^2 x_2^2$ .

Person 3's preferences can be represented by the utility function

$$U^3(x_1^3, x_2^3) = \min(x_1^3, x_2^3).$$

7. Prove (both)

(i) that every Walrasian (competitive) equilibrium allocation is in the core

(ii) that every allocation in the core is Pareto optimal

**continued**

8. What are all the Nash equilibria (in pure and mixed strategies) to the following game in strategic form?

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
I	(4, 2)	(3, 1)	(2, 0)	(2, 0)
II	(2, 3)	(0, 0)	(2, 3)	(1, 2)
III	(0, 2)	(4, 1)	(0, 1)	(2, 4)
IV	(0, 4)	(10, 2)	(0, 2)	(1, 1)

9. If there are three bidders in an auction, and each bidder's private value of the object being auctioned is an independent draw from the set of values  $\{1, 2, 3\}$ , with each of the 3 values equally likely,

- i* What is the expected revenue from an auction which allocates the object efficiently?
- ii* Design an auction which has a higher expected revenue than the efficient auction.

**continued**

10. What is the sub-game perfect Nash equilibrium to the following game?

There are two players in the game. Firm 1 is a prospective entrant, and firm 2 is an incumbent firm, which already has stores in two markets.

Firm 1 moves first, choosing whether to enter market A, or not to enter.

Firm 2 observes firm 1's first move. If firm 1 chose not to enter, firm 2 has no move to make. But if firm 1 chose (in the initial stage) to enter, then firm 2 chooses whether to accommodate entry or to start a price war in market A.

Firm 1 then chooses whether to enter market B. Firm 1 makes this choice immediately after choosing not to enter market A (if it chose not to enter in the first stage), or makes this choice after observing firm 2's move (if it had chosen to enter in the first stage).

If firm 1 chooses not to enter market B, the game ends. But if firm 1 chooses to enter market B, then firm 2 has a second move, whether to accommodate entry in market B, or to start a price war there. Then the game ends.

The firms' payoffs are the sum of their profits in the two markets.

Firm 1 gets profits of 0 in a market it does not enter, profits of 5 in a market which it entered and in which firm 2 accommodated its entry, and  $-2$  in a market which it entered and in which firm 2 started a price war.

Firm 2 gets profits of 10 in a market in which firm 1 did not enter, profits of 5 in a market in which firm 1 entered and in which it (firm 2) accommodated entry, and profits of  $-2$  in a market in which firm 1 entered and in which it (firm 2) started a price war.