GS/ECON 5010 APPLIED MICROECONOMICS

Answers to Midterm Exam October 2010

Q1. List (without proof) 5 properties of the indirect utility function of a consumer with well–behaved preferences.

A1. The text (pg. 28) lists 6 properties. Any 5 of them will do.

They are

1. the indirect utility function is continuous in prices and income

2. the indirect utility function is homogeneous of degree 0 in prices and income together : $v(a\mathbf{p}, ay) = v(\mathbf{p}, y)$ for any positive constant a

3. the indirect utility function is strictly increasing in income y (increasing y must shift out the budget line)

4. the indirect utility function is decreasing in prices (increasing p_i cannot increase utility, and must decrease it strictly if $x_i^M > 0$)

5. it's quasiconvex : if $v(\mathbf{p}, y) = v(\mathbf{p}', y')$, then $v(\mathbf{p}'', y'') \leq v(\mathbf{p}, y) = v(\mathbf{p}', y')$ if $\mathbf{p}'' = t\mathbf{p} + (1-t)\mathbf{p}'$ and $y'' = ty + (1-t)y$, for any $0 \leq t \leq 1$.

6. Roy's identity :

$$x_i^M(\mathbf{p}, y) = -\frac{\partial v(\mathbf{p}, y)/\partial p_i}{\partial v(\mathbf{p}, y)/\partial y}$$

for any good i

Q2. What are a person's Hicksian (compensated) demand functions, **and** her expenditure function, if her direct utility function is

$$u(x_1, x_2) = x_1 + \ln x_2 \qquad ?$$

A2. Solving directly from the expenditure minimization ("dual") problem, the consumer chooses x_1 and x_2 so as to minimimize $p_1x_1 + p_2x_2$ subject to the constraint $u(x_1, x_2) = u$. Here that minimization problem has a Lagrangean function

$$\mathcal{L} = p_1x_1 + p_2x_2 - \mu(x_1 + \ln x_2 - u)$$

with first–order conditions

$$p_1 = \mu \qquad (2-1)$$

$$p_2 = \frac{\mu}{x_2} \qquad (2-2)$$

Substituting for μ from $(2-1)$ into $(2-2)$ yields

$$p_2 = \frac{p_1}{x_2}$$

or

$$x_2 = \frac{p_1}{p_2} \qquad (2-3)$$

which is the Hicksian demand function for good 2.

Substituting for x_2 from $(2-3)$ into the utility constraint $x_1 + \ln x_2 = u$ yields

$$x_1 + \ln p_1/p_2 = x_1 + \ln p_1 - \ln p_2 = u$$

or

$$x_1 = u - \ln p_1 + \ln p_2 \qquad (2-4)$$

which is the Hicksian demand function for good 1.

Since

$$e(p_1, p_2, u) = p_1 x_1^H(p_1, p_2, y) + p_2 x_2^H(p_1, p_2, y)$$

here

$$e(p_1, p_2, y) = p_1 u - p_1 \ln p_1 + p_1 \ln p_2 + p_1 \qquad (2-5)$$

Differentiating $(2-5)$ with respect to p_1 and p_2 yields $(2-4)$ and $(2-3)$ respectively, so that Shephard's Lemma holds.

[The expenditure function can also be obtained using the "primal" problem. Maximizing utility subject to the budget constraint implies first–order conditions

$$1 = \lambda p_1$$

$$\frac{1}{x_2} = \lambda p_2$$

so that the Marshallian demand function for good 2 is

$$x_2^M(p_1, p_2, y) = \frac{p_1}{p_2} \qquad (2-6)$$

which is the same as the Hicksian demand function, since we have quasi–linear preferences here. Substituting from $(2-6)$ into the budget constraint yields the Marshallian demand function for good 1,

$$x_1^M(p_1, p_2, y) = \frac{y}{p_1} - 1 \qquad (2-7)$$

Substituting from $(2-6)$ and $(2-7)$ into the direct utility function, so that $v(p_1, p_2, y) = u[x_1^M(p_1, p_2, y), x_2^M(p_1, p_2, y)]$ implies that the indirect utility function here is

$$v(p_1, p_2, y) = \frac{y}{p_1} - 1 + \ln p_1 - \ln p_2 \qquad (2-8)$$

The duality relation

$$v(p_1, p_2, e(p_1, p_2, u)) = u$$

and equation $(2-8)$ then imply that

$$u = \frac{e(p_1, p_2, u)}{p_1} - 1 + \ln p_1 - \ln p_2 \qquad (2-9)$$

Equation $(2-9)$ can be re–arranged into expression $(2-5)$ for the expenditure function, and Shephard's Lemma then used to get the Hicksian demand functions $(2-4)$ and $(2-3)$.]

Q3. A risk–averse expected utility maximizer has a utility–of–wealth function

$$u(W) = \ln W$$

She has initial wealth of $1,000,000, half of which is invested in a house. There is a probability of 10 percent that her house will burn down this year and be destroyed totally, reducing her wealth to $500,000.

But she can buy insurance on her house. A firm is willing to sell her I dollars worth of insurance on the house, at an annual price of qI, where $q \geq 0.1$. [So she would collect I dollars from the insurance company if her house burned down, if she purchased a policy.]

She is free to choose to buy as much (or as little) insurance as she wishes.

How much insurance should she buy?

A3. If the person purchases I dollars worth of insurance, then her wealth will be $1,000,000 - qI$ in the "good" state of the world, in which her house does not burn down ; she must pay a price of q per dollar of insurance coverage chosen, so that qI is the total cost of her incurance.

In this case, in the "bad" state of the world she still has to pay qI for her insurance, but now she collects I dollars to partially compensate for the loss of the $500,000 house. So in the bad state her wealth will be

$$500,000 + I - qI$$

if she purchases I dollars of insurance.

Her expected utility is $(0.9)u(W_g) + (0.1)u(W_b)$, where W_g and W_b are her wealth in the good and bad states of the world. Here,

$$EU = (0.9) \ln (1,000,000 - qI) + (0.1) \ln (500,000 + I - qI) \qquad (3-1)$$

She should choose her insurance coverage I so as to maximize her expected utility defined by $(3-1)$. Setting the derivative of $(3-1)$ with respect to I equal to 0 yields the first–order condition

$$(0.1)\frac{1-q}{500,000 + (1-q)I} - (0.9)\frac{q}{1,000,000 - qI} = 0 \qquad (3-2)$$

Equation $(3-2)$ can be written

$$9q[500,000 + (1-q)I] = (1-q)[1,000,000 - qI] \qquad (3-3)$$

or

$$I = \frac{(2 - 11q)}{q(1-q)} 50,000 \qquad (3-4)$$

which is the amount of coverage she should buy.

Note that if the insurance is actuarially fair, then $q = 0.1$: in this case the expected payout from the policy $(0.1)I$ equals the premium paid, qI. When $q = 0.1$, equation $(3-4)$ says that the person buys full coverage, \$500,000, so that $W_b = 500,00 + (1-q)I = 950,000 = W_g$.

If insurance is really expensive ($q > 0.1819$), then the right side of equation $(3-4)$ is negative. Even though the person is risk averse, she will choose not to buy any insurance if insurance is too expensive ; going without insurance increases her expected wealth, so that she is willing to take that bet if the insurance is priced too high.

When $q < 0.1819$, differentiation of $(3-4)$ with respect to q shows that the amount of insurance I that she chooses to buy is a decreasing function of the price q.