

time=2.5 hours

Do any **6** of the following 10 questions. All count equally.

1. State and prove the Slutsky equation, relating the derivatives of Marshallian demand functions with the derivatives of Hicksian demand functions.

2. How much insurance I would a person choose to purchase, against a total loss of L , if the probability of the loss were π , her initial wealth was $W > L$, and her utility-of-wealth function was

$$U(W) = \ln(W + A)$$

where A is a positive constant, and she could buy as little or as much insurance coverage I as she wished, at a premium of p per dollar of coverage?

3. What is a firm's cost function, and its conditional input demand functions, if its production function is

$$y = 100 + x_1 - \frac{100}{1 + x_2}$$

where y is the quantity of output, and x_1 and x_2 are the quantities used of two inputs?

4. What is the relation between a monopoly's price, and its marginal cost, if it must sell all its output at the same unit price?

continued

5. What are the Cournot–Nash equilibria in a duopoly in which firms choose simultaneously the quantities to produce of a homogeneous good, if the demand function for the good has the equation

$$Q = 130 - p$$

and each firm's total cost function has the equation

$$\begin{aligned} TC(q) &= 2000 + 10q & q > 0 \\ &= 0 & q = 0 \end{aligned}$$

where Q is total industry output, and p is the price?

6. Give an example of an exchange economy in which some feasible allocation is **not** in the core, even though it is Pareto optimal, and preferred by each person to the person's initial endowment.

7. Find a competitive allocation in the following exchange economy.

There are 2 million people, each with an endowment vector $\mathbf{e}^h = (1, e_2)$ of the two goods. One million people have preferences which can be represented by the utility function

$$U^1(x_1^1, x_2^1) = x_1^1 + \ln(x_2^1)$$

and the other one million people have preferences which can be represented by the utility function

$$U^2(x_1^2, x_2^2) = (x_1^2)(x_2^2)$$

continued

8. Write down the strategic form of the following game, and find all its Nash equilibria in pure strategies.

There are two firms, each choosing simultaneously the **quantities** to produce of a homogeneous good. The aggregate demand function for the good is

$$Q = 6 - p$$

where Q is the aggregate quantity demanded and p the market price.

Firms choose quantities simultaneously, with the market price determined from the market demand function specified above. Firms want to maximize their profits.

Each firm incurs a fixed cost of production $F = 3$ if it produces any output at all. If the firm produces an output level of 0, it incurs no costs. If it pays the fixed cost $F = 3$, it can produce any quantity $q \leq 3$, at the same total cost (of 3). (So marginal costs of production are 0.)

Firms' quantity choices must be non-negative integers, and these quantities must be less than or equal to 3.

9. The extensive form game shown on the next page is one in which nature moves first, player 1 moves next, after observing nature's move, and then player 2 moves, after observing player 1's move, but not nature's move.

Is there a sequential equilibrium to this game in which player 1 always plays B if nature chose "Right", and in which player 1 chooses B with probability $1/4$ if nature chose "Left"?

Explain briefly.

10. Show that the symmetric Nash equilibrium in a first-price sealed bid auction, with each bidder's private valuation of the object an independent draw from the uniform distribution, is for a bidder with value v to place a bid of $\frac{n-1}{n}v$ on the object, if there are n bidders.

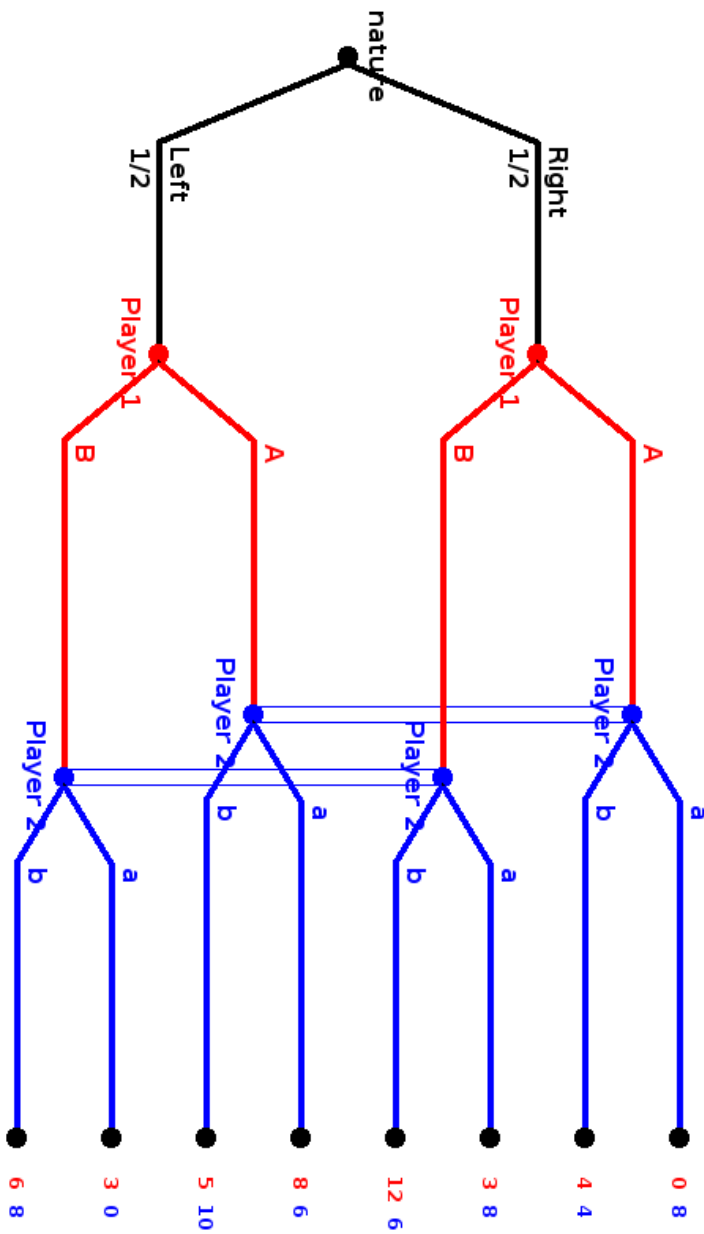


Figure (Question 9) : extensive form game for question # 9