GS/ECON 5010 section "B" APPLIED MICROECONOMICS

Answers to Midterm Exam October 2011

*Q*1. What are the Marshallian (uncompensated) demand functions for a consumer whose preferences can be represented by the utility function

$$u(x_1, x_2) = 100 - \frac{1}{x_1} - \frac{4}{x_2} \qquad ?$$

*A*1. The first–order conditions for utility maximization by consumers are

$$u_1 = \frac{1}{(x_1)^2} = \lambda p_1 \tag{1-1}$$

$$u_2 = \frac{4}{(x_2)^2} = \lambda p_2 \tag{1-2}$$

so that dividing $(1-1)$ by $(1-2)$ yields

$$\frac{(x_2)^2}{4(x_1)^2} = \frac{p_1}{p_2} \tag{1-3}$$

or

$$x_2 = 2\sqrt{\frac{p_1}{p_2}}x_1 \tag{1-4}$$

Substitution of $(1-4)$ into the budget constraint

$$y = p_1 x_1 + p_2 x_2 \tag{1-5}$$

yields

$$y = p_1 x_1 + 2\sqrt{p_1 p_2}x_1 \tag{1-6}$$

so that

$$x_1 = \frac{1}{\sqrt{p_1}} \frac{y}{\sqrt{p_1} + 2\sqrt{p_2}} \tag{1-7}$$

which is the Marshallian demand function for good #1. From equation $(1-4)$, then, the Marshallian demand function for good #2 is

$$x_2 = \frac{2}{\sqrt{p_2}} \frac{y}{\sqrt{p_1} + 2\sqrt{p_2}} \tag{1-8}$$

$Q2$. How much would a risk–averse expected utility maximizer be willing to pay for an insurance policy which offers complete coverage against a loss of L if her initial wealth were W, the probability of the loss were π, and her utility–of–wealth function were

$$U(W) = \ln W \qquad ?$$

$A2$. If the person does not purchase insurance, her expcted utility is

$$EU = \pi \ln(W - L) + (1 - \pi) \ln W \tag{2-1}$$

and if she purchases full insurance at a price of P, then her wealth will be $W - P$, whether of not the loss happens, so that her expected utility is

$$U^I = \ln(W - P) \tag{2-2}$$

The highest price she would be willing to pay for full insurance is the price P which makes EU equal to U^I, so that

$$\ln(W - P) = \pi \ln(W - L) + (1 - \pi) \ln W \tag{2-3}$$

Taking exponents of both sides and using the fact that $e^{(a+b)} = e^a e^b$,

$$e^{\ln(W-P)} = e^{\pi \ln(W-L)} e^{(1-\pi)\ln(W)} \tag{2-4}$$

Since $e^{a \ln b} = b^a$, therefore

$$W - P = (W - L)^{\pi} W^{1-\pi} \tag{2-5}$$

so that

$$P = W - (W - L)^{\pi} W^{1-\pi} \tag{2-6}$$

[It can be checked that this risk averse person is willing to pay a premium for insurance : that is, the price P she is willing to pay will exceed the expected loss πL, whenever $0 < \pi < 1$.

From equation $(2-3)$,

$$\frac{\partial P}{\partial \pi} = (W - P)(\ln W - \ln (W - P)) > 0 \tag{2-7}$$

and

$$\frac{\partial^2 P}{\partial \pi^2} = -\frac{\partial P}{\partial \pi}(\ln W - \ln (W - P)) < 0 \tag{2-8}$$

Hence P is a strictly concave function of the probability π of a loss. That means that $P - \pi L$ is also a strictly concave function of π. At $\pi = 0$, $P = 0 = \pi L$, and at $\pi = 1$, $P = L = \pi L$. So the function $f(\pi) = P - \pi L$ equals 0 at $\pi = 0$, equals 0 at $\pi = 1$, and is strictly concave. That means that the function must be positive for $0 < \pi < 1$, so that $P > \pi L$.]

*Q*3. Explain why perfect competition is inconsistent with increasing returns to scale.

*A*3. A couple of different explanations :

(*i*) Profit maximization under perfect competition implies that each factor be paid the value of its marginal product, so that $pf_i = w_i$ where p is the output price, f_i the marginal product of input i and w_i the price of input i.

The definition of the local measure of scale economices $\mu(\mathbf{x})$ is that

$$\mu(\mathbf{x}) = \frac{\sum_i f_i x_i}{f(\mathbf{x})} \tag{3-1}$$

so that

$$\mu(\mathbf{x}) = \frac{\sum w_i x_i}{pf(\mathbf{x})} \tag{3-2}$$

when the firm maximizes its profit in perfect competition. Therefore, the firm's costs $\sum_i w_i x_i$ will exceed the firm's revenue $pf(\mathbf{x})$ if the firm operates under conditions of increasing returns to scale ($\mu(\mathbf{x}) > 1$).

(ii) Suppose that the competitive firm's profit maximization problem has a well–defined solution, in which the firm uses the input combination $\mathbf{x}^* \neq 0$, and earns profits of $\pi^* = pf(\mathbf{x}^*) - \mathbf{w} \cdot \mathbf{x}^*$. Since the firm always has the option of shutting down and making zero profits, therefore $\pi^* \geq 0$. Increasing returns to scale implies then that if the firm doubled all its inputs, it would make profits of

$$\pi^{**} = pf(2\mathbf{x}^*) - \mathbf{w} \cdot 2\mathbf{x}^* = pf(2\mathbf{x}^*) - 2\mathbf{w} \cdot \mathbf{x}^* > 2pf(\mathbf{x}^*) - 2\mathbf{w} \cdot \mathbf{x}^* = 2\pi^* \geq \pi^* \quad (3-3)$$

under increasing returns to scale. Since $\pi^{**} > \pi^*$, the original solution could not have been an optimum. So there cannot be a well–defined solution to the firm's profit maximization problem.

(iii) This third explanation is only true if the firm's production function is homogeneous of degree α.

The firm's profit maximization problem in perfect competition is to maximize $py - C(\mathbf{w}, y)$ with respect to y. The first–order condition for a profit maximum is

$$p - \frac{\partial C(\mathbf{w}, y)}{\partial y} \quad (3-4)$$

and its second–order condition is

$$\frac{\partial^2 C(\mathbf{w}, y)}{\partial y^2} \geq 0 \quad (3-5)$$

which implies that the firm's **marginal** cost $\frac{\partial C(\mathbf{w},y)}{\partial y}$ must be non–decreasing.

Increasing returns to scale imply that the firm's **average** cost $\frac{C(\mathbf{w},y)}{y}$ be decreasing.

But, in general, it may be possible for a firm to have increasing marginal cost, even if it operates under increasing returns to scale.

However, if the firm's production is homogeneous of degree α, then

$$C(\mathbf{w}, y) = y^{1/\alpha} C(\mathbf{w}, 1) \quad (3-6)$$

so that

$$\frac{\partial^2 C(\mathbf{w}, y)}{\partial y^2} \geq 0 \quad \text{if} \quad \text{and} \quad \text{only} \quad \text{if} \quad \alpha \leq 1$$

which is exactly the condition for the firm not to have increasing returns to scale.

[If the firm's production function is not homogeneous of degree α, then the cost function could satisfy the second–order conditions, and still exhibit increasing returns to scale. For example if

$$C(\mathbf{w}, y) = 24\frac{y}{y+1} + 4y^2 - 13y \qquad y \leq 1$$

$$C(\mathbf{w}, y) = 2y + \frac{1}{y} \qquad y > 1$$

for some input price vector $\mathbf{w}$, then the cost function would be continuously differentiable at $y = 1$, and would have $MC' > 0$ whenever $y > 1$, but it would exhibit increasing returns to scale whenever $y > 1$, since average cost decreases with output whenever $y > 1$.]