GS/ECON 5010 section "B" APPLIED MICROECONOMICS

Answers to Midterm Exam October 2012

$Q1$. Derive the Slutsky equation, which shows the relationship between the derivatives of the Marshalllian and Hicksian demand functions.

$A1$. The definitions of the Hicksian and Marshallian demand functions imply that

$$\mathbf{x}^M(\mathbf{p}, e(\mathbf{p}, u)) = \mathbf{x}^H(\mathbf{p}, u) \qquad (1-1)$$

where $e(\mathbf{p}, u)$ is the expenditure function.

Differentiating the above expression with respect to some price p_j,

$$\frac{\partial x_i^M}{\partial p_j} + \frac{\partial x_i^M}{\partial y}\frac{\partial e(\mathbf{p}, u)}{\partial p_j} = \frac{\partial x_i^H}{\partial p_j} \qquad (1-2)$$

for any good i.

Shepherd's Lemma says that

$$\frac{\partial e(\mathbf{p}, u)}{\partial p_j} = x_j^H(\mathbf{p}, u) \qquad (1-3)$$

and the definition of Marshallian and Hicksian demand functions says that

$$x_j^H(\mathbf{p}, u) = x_j^M(\mathbf{p}, y) \qquad (1-4)$$

when $y = e(\mathbf{p}, u)$. So $(1-3)$ and $(1-4)$ imply that $(1-2)$ can be written

$$\frac{\partial x_i^M}{\partial p_j} + \frac{\partial x_i^M}{\partial y} x_j^M(\mathbf{p}, y) = \frac{\partial x_i^H}{\partial p_j} \qquad (1-5)$$

Moving the second term on the left side of $(1-5)$ to the right gives us the Slutsky equation

$$\frac{\partial x_i^M}{\partial p_j} = \frac{\partial x_i^H}{\partial p_j} - \frac{\partial x_i^M}{\partial y} x_j^M(\mathbf{p}, y) \qquad (1-6)$$

$Q2$. If someone is risk averse (but we do not know how risk averse), what are the possible values for this person's risk premium for a gamble

$$g = (0.5 \circ 40, 0.5 \circ 80) \quad ?$$

$A2$. The risk premium is the difference between the expected value of the gamble, and the certainty equivalent ot the gamble. Here the expected value of the gamble is 60. The certainty equivalent to the gamble must be less than 60, if the person is risk averse.

But the certainty equivalent must be at least 40 : a risk–averse expected utility maximizer would always prefer a gamble which yields at least 40, with a chance at more than 40, to getting 40 for sure.

Since CE is between 40 and 60, the risk premium P must be between 0 and 20 (since $P = 60 - CE$).

And that's all we can say about P, if we just know that the person is risk averse.

[For example, if the person's preferences exhibited a constant coefficient of relative risk aversion β, then the certainty equivalent would be defined by

$$(0.5)40^{1-\beta} + (0.5)80^{1-\beta} = CE^{1-\beta}$$

which implies that

$$CE = 2^{-1/(1-\beta)}[1 + 2^{1-\beta}]^{1/(1-\beta)}40$$

When $\beta = 0$, the above expression equals 60, and as $\beta \to \infty$, the expression above approaches 40.]

*Q*3. What is the cost function $C(w_1, w_2, y)$ for a firm with a production function

$$f(x_1, x_2) = \frac{1}{2}(\ln(x_1 + 1) + \ln(x_2 + 1))$$

where "ln" is the natural logarithm?

*A*3. When the production function is

$$f(x_1, x_2) = \frac{1}{2}(\ln(x_1 + 1) + \ln(x_2 + 1))$$

then the marginal products of the 2 inputs are

$$f_1 = \frac{1}{2(x_1 + 1)} \qquad (3-1)$$

$$f_2 = \frac{1}{2(x_2 + 1)} \qquad (3-2)$$

Cost minimization by the firm requires to choose an input mix for which

$$\frac{f_1}{f_2} = \frac{w_1}{w_2} \qquad (3-3)$$

Here, this means (from $(3-1)$ and $(3-2)$) that

$$x_2 + 1 = \frac{w_1}{w_2}(x_1 + 1) \qquad (3-4)$$

If the output level is y, then $(3-4)$ implies that

$$y = \frac{1}{2}[(\ln(x_1 + 1) + \ln(\frac{w_1}{w_2}[x_1 + 1])] \qquad (3-5)$$

Using the property that $\ln(ab) = \ln a + \ln b$, equation $(3-5)$ can be written

$$y = \ln(x_1 + 1) + \frac{1}{2}\ln w_1 - \frac{1}{2}\ln w_2 \qquad (3-6)$$

or

$$x_1 = \sqrt{\frac{w_2}{w_1}}e^y - 1 \qquad (3-7)$$

which is the conditional input demand for input 1. Substituting from $(3-7)$ into $(3-4)$, the conditional input demand for input 2 is

$$x_2(w_1, w_2, y) = \sqrt{\frac{w_1}{w_2}e^y} - 1 \qquad (3-8)$$

The cost function $C(w_1, w_2, y)$ is defined as $w_1 x_1(w_1, w_2, y) + w_2 x_2(w_1, w_2, y)$ so that here

$$C(w_1, w_2, y) = 2\sqrt{w_1 w_2 e^y} - w_1 - w_2 \qquad (3-9)$$

[Notice that the expression defined by $(3-9)$ is increasing in w_1, w_2 and y, is homogeneous of degree 1 in (w_1, w_2), is concave, and that its derivatives with respect to w_1 and w_2 are the right sides of expressions $(3-7)$ and $(3-8)$ respectively.]