GS/ECON 5010 section "B" APPLIED MICROECONOMICS

Answers to Midterm Exam October 2013

$Q1$. What is a consumer's expenditure function, if her (direct) utility function is

$$U(x_1, x_2) = \log(x_1) + \log(x_2)$$

(where "log" denotes the natural logarithm, and where $0 < a < 1$ is a constant)?

$A1$. These are Cobb–Douglas preferences (which means that they are a special case of *CES* preferences, with an elasticity of substitution equal to 1).

Solving directly, the first–order conditions for the minimization of $p_1x_1 + p_2x_2$ subject to $U(x_1, x_2) = u$ are

$$p_1 = \frac{\mu}{x_1} \quad (1-1)$$

$$p_2 = \frac{\mu}{x_2} \quad (1-2)$$

where μ is the Lagrange multiplier on the constraint $\log(x_1) + \log(x_2) = u$.

Equations $(1-1)$ and $(1-2)$ imply that

$$x_2 = \frac{p_1}{p_2}x_1 \quad (1-3)$$

so that the utility constraint implies that

$$\log(x_1) + \log(\frac{p_1x_1}{p_2}) = u \quad (1-4)$$

Using the fact that $\log(\frac{\alpha\beta}{\gamma}) = \log(\alpha) + \log(\beta) - \log(\gamma)$, equation $(1-4)$ implies that

$$2\log(x_1) + \log(p_1) - \log(p_2) = u \quad (1-5)$$

or

$$x_1 = e^{u/2}(\frac{p_2}{p_1})^{1/2} \quad (1-6)$$

which is the Hicksian demand function for good 1. Substituting from equation $(1-3)$ into $(1-6)$,

$$x_2 = e^{u/2}(\frac{p_1}{p_2})^{1/2} \qquad (1-7)$$

Since $e(\mathbf{p}, u) = x_1^H(\mathbf{p}, u) + p_2^H(\mathbf{p}, u)$, therefore, the expenditure function is

$$e(\mathbf{p}, u) = 2e^{u/2}[p_1 p_2]^{1/2} \qquad (1-8)$$

Alternatively, we can start with the Marshallian demand functions for Cobb–Douglas preferences,

$$x_i^M(\mathbf{p}, y) = \frac{y}{2p_i} \quad i = 1, 2 \qquad (1-9)$$

and plug them into the definition of the direct utility function, so that

$$v(\mathbf{p}, y) = \log{(\frac{y}{2p_1})} + \log{(\frac{y}{2p_2})} = 2\log y - 2\log 2 - \log{(p_1)} - \log{(p_2)} \qquad (1-10)$$

And use the fact that $v(\mathbf{p}, e(\mathbf{p}, u)) = u$ to infer that

$$2\log{[e(\mathbf{p}, u)]} - 2\log 2 - \log{(p_1)} - \log{(p_2)} = u \qquad (1-11)$$

implying that

$$\log{[e(\mathbf{p}, u)]} = \frac{u}{2} + \log 2 + \frac{1}{2}\log{(p_1)} + \frac{1}{2}\log{(p_1)} \qquad (1-12)$$

Taking anti–logarithms of both sides of equation $(1-12)$ yields equation $(1-8)$, the expression for the expenditure function.

$Q2$. (Without proof), give two different properties which are equivalent to the statement : "person 1, with the utility–of–wealth function $U(W)$ is always more risk averse than person 2, with the utility–of–wealth function $V(W)$".

$A2$. Pages 112 – 115 of *Jehle and Reny* mention these properties. In no particular order, the following statements are equivalent to the statement in the question :

i Any gamble which person 1 is willing to take, person 2 is also willing to take.

ii. For any gamble g, the **certainty equivalent** CE_1 for person 1, defined by

$$EU(g) = U(CE_1)$$

is smaller than the certainty equivalent for person 2, defined by

$$EV(g) = V(CE_2)$$

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iia. For any gamble g, the risk premium RP for the gamble, defined by

$$RP = Eg - CE$$

is higher for person 1 than for person 2.

iii. Person 1's utility function is **more concave** than person 2's : there exists an increasing, concave function $h(\cdot)$ such that

$$U(W) = h[V(W)]$$

iv. For any level of wealth W, person 1's coefficient of absolute risk aversion, defined by

$$R_A^1(W) \equiv -\frac{U''(W)}{U'(W)}$$

is greater than person 2's coefficient of absolute risk aversion

$$R_A^2(W) \equiv -\frac{V''(W)}{V'(W)}$$

iva. For any level of wealth W, person 1 has a greater coefficient of relative risk aversion than person 2, where the coefficient of relative risk aversion is defined by

$$R_R^i(W) \equiv R_A^i(W)W$$

*Q*3. What is the profit function $\pi(p, w_1, w_2)$ for a firm with a cost function

$$C(w_1, w_2, y) = \frac{w_1 w_2}{(w_1 + w_2)^2} y^2 \qquad ?$$

*A*3. The firm's profit function $\pi(p, w_1, w_2)$ is the maximum value of

$$py - C(w_1, w_2, y) \tag{3 – 1}$$

with respect to y.

In this case, the firm chooses y to maximize

$$py - \frac{w_1 w_2}{(w_1 + w_2)^2} y^2 \tag{3 – 2}$$

yielding a first–order condition

$$2y \frac{w_1 w_2}{(w_1 + w_2)^2} = p \tag{3 – 3}$$

or

$$y(p, w_1, w_2) = \frac{p(w_1 + w_2)^2}{2 w_1 w_2} \tag{3 – 4}$$

which is the firm's supply function. [The second–order condition for profit maximization is satisfied here : the second derivative of $(3-1)$ with respect to y is $-2\frac{w_1 w_2}{(w_1+w_2)^2} < 0$.]

Plugging $(3-4)$ into the definition $(3-1)$ of profit

$$\pi(p, w_1, w_2) = \frac{p^2(w_1 + w_2)^2}{2 w_1 w_2} - \frac{w_1 w_2}{(w_1 + w_2)^2} \frac{p^2(w_1 + w_2)^4}{4(w_1 w_2)^2} = \frac{p^2(w_1 + w_2)^2}{4 w_1 w_2} \tag{3 – 5}$$