

time=2.5 hours

Do any **6** of the following 10 questions. All count equally.

1. If a person's preferences can be represented by the utility function

$$u(x_1, x_2, x_3) = x_1 + \log x_2 + A \log x_3$$

(where "log" denotes the natural logarithm, and A is a positive constant), find the person's Marshallian demand functions for each good, her indirect utility function, her Hicksian demand functions, and her expenditure function.

2. How much insurance I would a person choose to buy, if she could buy as much or as little insurance as she wished, at a price of p per dollar of insurance coverage, in the following situation?

She has initial wealth of W , she expects to lose L dollars with probability π , she will collect I dollars from the insurance company if this loss (of L dollars) occurs (if she has purchased I dollars of insurance coverage), and she is a von Neumann–Morgenstern expected utility maximizer, with utility-of-wealth function

$$U(W) = \frac{1}{1-\beta} W^{1-\beta}$$

where $\beta > 0$.

3. Prove Shephard's Lemma, that the derivative of a firm's cost function with respect to the price of an input is the firm's conditional demand for the input.

4. What is the equation of the long-run industry supply curve of a perfectly competitive industry in which there are a large number of identical firms, each of which has the same total cost function

$$TC(y) = y^3 - 24y^2 + 200y$$

where $TC(y)$ is the total cost of producing y units of output?

5. Show how the total industry profit varies with the number n of firms, in an industry with all of the following feature :

(i) each firm produces the same homogeneous good

(ii) the total quantity demanded of the good by buyers obeys the demand function $Q = a - p$ where Q is total quantity demanded and p the price

(iii) firms choose their quantities simultaneously and non-cooperatively

(iv) each firm can produce as much or as little as it wishes, at a constant marginal cost of c per unit produced

6. Is the allocation $\mathbf{x}^1 = (2.5, 2.5)$, $\mathbf{x}^2 = (1, 1)$, $\mathbf{x}^3 = (2.5, 2.5)$ in the core of the 3-person exchange economy described below? Explain why or why not.

Person 1 regards the two goods as **perfect substitutes**.

Person 2 and person 3 each regard the two goods as **perfect complements**.

The endowments of the three people are $\mathbf{e}^1 = (3, 0)$, $\mathbf{e}^2 = (3, 0)$, $\mathbf{e}^3 = (0, 6)$.

continued

7. Prove (both)

(i) that every Walrasian (competitive) equilibrium allocation is in the core

(ii) that every allocation in the core is Pareto optimal

8. What are all the Nash equilibria (in pure and mixed strategies) to the following game in strategic form?

	<i>a</i>	<i>b</i>	<i>c</i>	<i>d</i>
I	(3, 5)	(4, 3)	(2, 8)	(0, 4)
II	(8, 7)	(8, 2)	(1, 0)	(6, 2)
III	(0, 0)	(4, 0)	(6, 3)	(8, 5)
IV	(6, 3)	(12, 6)	(4, 4)	(5, 6)

9. *for question 9, see the next page*

10. What is the expected revenue from a first-price sealed bid auction (or from any other efficient auction) if there are two bidders, and each bidder's valuation of the object is an independent draw from the uniform distribution over $[0, 1]$?

continued

9. Write down the strategic form diagram for the following game of imperfect information, and find a perfect Bayesian equilibrium (or a sequential equilibrium) to the game, in which player 1 (“job candidate”) chooses the same action, regardless of what nature’s move was (i.e. find an equilibrium in which player 1’s action does not reveal what nature’s move was).

Nature moves first, choosing whether the job candidate will be “good” or “bad”. The *ex ante* probability with which nature chooses the outcome “good” is 0.75.

Player 1 observes the actual realization of nature’s move (whether she herself is “good” or “bad”). Player 2 does not observe nature’s move, but knows the *ex ante* probabilities.

Player 1 moves first, choosing whether or not to get a diploma. It costs player 1 (the job candidate) \$1 to get a diploma if she is good, and \$3 to get a diploma if she is bad.

Player 2, the “hiring committee”, observes player 1’s move (whether or not she has a diploma), and then chooses whether or not to hire the job candidate. If player 2 does not hire player 1, then player 2 gets a payoff of 0, and player 1 gets a payoff of 0, minus any diploma costs she may have incurred. If the hiring committee chooses to hire the job candidate, then the job candidate gets a payoff of 5 (minus any diploma costs she may have incurred), and the hiring committee gets a payoff of 2 if the job candidate is good, and -2 if the job candidate is bad.