Solution Concepts for Games in Strategic Form

solution?

prediction of what happens in the game

so far...

games with dominant strategy

example : Prisoners' Dilemma (e.g. 3) : t is a strictly dominant strategy for player 1, L is a strictly dominant strategy for player 2

$$\begin{pmatrix} 1\backslash 2 & L & R \\ \\ t & (2,2) & (7,0) \\ b & (0,7) & (6,6) \end{pmatrix}$$

so that the solution is (t, L)

but in example 4, only one player (player #1) has a dominant strategy

$$\begin{pmatrix} 1\backslash 2 & L & R \\ & & \\ t & (2,4) & (7,0) \\ b & (0,0) & (6,6) \end{pmatrix}$$

so it cannot be solved using (just) dominant strategies

example 5 can be solved using weak dominance : t is a weakly dominant strategy for player 1, and L is a weakly dominant strategy for player 2

$$\begin{pmatrix} 1\backslash 2 & L & R \\ & & \\ t & (5,5) & (0,0) \\ b & (0,0) & (0,0) \end{pmatrix}$$

Elimination of Dominated Strategies

if a player has a (strictly) dominated strategy, then we can cross it out : why would she ever choose to play it?

so that a game like example 6, in which b is strictly dominated by t for player 1

$$\begin{pmatrix} 1\backslash 2 & L & R \\ & & \\ t & (5,5) & (8,0) \\ m & (8,4) & (-1,3) \\ b & (0,3) & (7,7) \end{pmatrix}$$

becomes example 7

$$\begin{pmatrix} 1\backslash 2 & L & R \\ & & \\ t & (5,5) & (8,0) \\ m & (8,4) & (-1,3) \end{pmatrix}$$

now R is a strictly dominated strategy for player 2 in example 7 (but it wasn't strictly dominated in example 6)

thought process (by player 2) : if player 1 were to play b, then R would be better for me than L ; but player 1 will **never** play b, because it is strictly dominated for her ; knowing that player 1 will never play b, I know that I should never play R

so we can cross out column R for player 2, which leaves us with a 1–by–2 game

$$\begin{pmatrix} 1\backslash 2 & L \\ & \\ t & (5,5) \\ m & (8,4) \end{pmatrix}$$

in which m is player 1's best strategy

so example 6 can be solved by **iterated elimination of strictly dominated strategies** ; the solution is (m, L)

1's thought process : I should never play b, since it is a strictly dominated strategy for me ; but player 2 knows the game, and he can see that b is strictly dominated for me ; so he knows that I will not play b, and therefore he concludes that he should not play R ; if he is not going to play R, then I should pick m

general definition : a game is solvable by interated elimination of strictly dominated strategies, if the process of crossing out strictly dominated rows and/or columns leads to only one row and one column left does the order of crossing out matter? not if the crossed–out strategies are strictly dominated

game 4 can also be solved by iterated elimination of strictly dominated strategies, while game 5 can be solved by iterated elimination of weakly dominated strategies

examples 1 and 2 cannot be solved by iterated elimination of strictly dominated strategies

example 8 can be solved by iterated elimination of weakly dominated strategies : but notice that the solution involves a pretty long chain of ”I know that she knows that I know that she knows ...”

common knowledge : players all know the game ; players know that the other players know the game ; players know that the other players know that they know the game ; etcetera

Example 8

$$\begin{pmatrix} 1\backslash 2 & L & CL & CR & R \\ \\ t & (1,0) & (1,0) & (1,0) & (1,0) \\ mt & (0,2) & (2,1) & (2,1) & (2,1) \\ mb & (0,2) & (1,3) & (3,2) & (3,2) \\ b & (0,2) & (1,3) & (2,4) & (4,3) \end{pmatrix}$$

column R is weakly dominated for player 2 ; cross that out to get

$$\begin{pmatrix} 1\backslash 2 & L & CL & CR \\ \\ t & (1,0) & (1,0) & (1,0) \\ mt & (0,2) & (2,1) & (2,1) \\ mb & (0,2) & (1,3) & (3,2) \\ b & (0,2) & (1,3) & (2,4) \end{pmatrix}$$

in which row b is weakly dominated for player 1, so that we get

$$\begin{pmatrix} 1\backslash 2 & L & CL & CR \\ \\ t & (1,0) & (1,0) & (1,0) \\ mt & (0,2) & (2,1) & (2,1) \\ mb & (0,2) & (1,3) & (3,2) \end{pmatrix}$$

now column CR is weakly dominated (by CL) for player 2, so we get

$$\begin{pmatrix} 1\backslash 2 & L & CL \\ \\ t & (1,0) & (1,0) \\ mt & (0,2) & (2,1) \\ mb & (0,2) & (1,3) \end{pmatrix}$$

now mb can be crossed out, and then CL, so that the solution is (t, L)

One More Extension..

a strategy is also strictly dominated if some **convex combination** of other strategies always does better....even if it is not strictly dominated by any single ("pure") strategy

as in game $8a$

$$\begin{pmatrix} 1\backslash 2 & L & R \\ \\ t & (3,8) & (2,4) \\ m & (2,0) & (6,3) \\ b & (8,2) & (0,4) \end{pmatrix}$$

in this game, no single strategy strictly dominates any other strategy (for either player)

but a convex combination : play row b half the time, and play row m half the time, leads to a row with expected payoffs 5 and 3 for player 1 : so t is dominated strictly by the **mixed** strategy : m with probability 0.5 and b with probability 0.5

so this game is solvable by iterated elimination of strictly dominated strategies, and has a solution (m, R)