Games in Extensive Form

the extensive form of a game is a "tree" diagram

except that my trees grow sideways

any game can be represented either using the extensive form or the strategic form

but the extensive form is probably the most useful tool to analyze games with a sequence of moves

the extensive form consists of **nodes** connected by **branches**

the nodes are the decision points of the game : points at which players choose actions

the branches correspond to the actions

so if player 1, at some point in the game, has a set of 7 actions from which to choose, then there will be 7 branches coming out of that node the **terminal nodes** at the bottom (or right) of the tree, represent points at which there are no more moves, and payoffs get collected

the initial node is the starting point

at each node (except the terminal nodes), some player is making a move

so that the node can be labelled with the name of the player making the move

Extensive Form Game 1

has 2 players, each making 1 move

player 1 moves first, choosing a or b

then player 2 moves, **after** he has observed player 1's move ; whatever player 1 has done, player 2 has 2 actions, A or B

here player 2 has only 2 actions (at either of his decision nodes), but he has 4 possible strategies :

i choose A no matter what ...

ii choose A if 1 chose a, choose B if 1 chose b

iii choose B if 1 chose a, choose A if 1 chose b

iv choose B no matter what ...

so that this game can also be represented in strategic form, by example 12

$$
\begin{pmatrix}
1\backslash 2 & aA, bA & aA, bB & aB, bA & aB, bB \\
 & & & & \\
a & (1,2) & (1,2) & (3,3) & (3,3) \\
b & (0,8) & (6,4) & (0,8) & (6,4)
\end{pmatrix}
$$

this game can be solved by iterated elimination of weakly dominated strategies

its unique Nash equilibrium is $(a, [aB, bA])$

Extensive Form Game 2

(changing only the top terminal node from extensive form game 1)

in strategic form, it's example 13

$$\begin{pmatrix} 1\backslash 2 & aA, bA & aA, bB & aB, bA & aB, bB \\ & & & & \\ a & (-5, 2) & (-5, 2) & (3, 3) & (3, 3) \\ b & (0, 8) & (6, 4) & (0, 8) & (6, 4) \end{pmatrix}$$

this game is also solvable by iterated elimination of weakly dominated strategies : $(a, [aB, bA])$

but it has another pure strategy Nash equilibrium, $(b, [aA, bA])$ (which involves player 1 playing a weakly dominated strategy)

this second Nash equilibrium seems to be a little implausible, if we look at the extensive form :

player 1 plays b because she thinks that player 2 would play A if she played a

but if she really did play a, why would player 2 pick an action yielding him 2, instead of B which yields him 3?

Extensive Form Game 3 : Entry Deterrence

the story :

player 2 has a store in some market (already there before the game starts)

player 1 moves first, deciding whether or not to open a competing store in firm 2's market ("enter" or "don't enter")

if player 1 chooses not to enter, the game ends

if player 1 chose to enter, player 2 chooses whether to start a (mutually destructive) price war, or to "accommodate" entry by keeping its prices high in strategic form, it is example 14

$$\begin{pmatrix} 1\backslash 2 & PW & A \\ & & \\ no & (0,10) & (0,10) \\ enter & (-2,-2) & (5,5) \end{pmatrix}$$

this game (again) can be solved by iterated limination of weakly dominated strategies : $(enter, A)$

but it also has another pure strategy Nash equilibrium, (no, PW), which involves player 2 playing a weakly dominated strategy

implausible : the threat of a price war keeps firm 1 from entering, but it should realize that firm 2 would not find if in its own interest to start a price war if entry actually occurred

Sub–Game Perfect Nash Equilibrium [SPNE]

a **sub–game** of any game in extensive form is simply all the game starting from any node in the game

warning : this will be modified soon

each sub–game can be treated as a game on its own

the strategies for any sub–game are just the strategies for the original game, restricted to the sub–game

so we can find Nash equilibria to each sub–game

a pair of strategies is a SPNE for a game in extensive form if the strategies comprise a Nash equilibrium to **every** sub–game of the game

SPNE for Extensive Form Game 3

the only sub–game to this game (other than the game itself) is the one starting at the node $enter$

the Nash equilibrium to that little game is A : it's a "one player" game, and player 2 will pick the strategy which gives him the highest payoff

so (no, PW) is **not** a SPNE to this game, since PW is not a Nash equilibrium to the sub–game

$(enter, A)$ is a SPNE

Theorems 7.4, 7.5

every (finite, perfect information) extensive form game has an SPNE

the SPNE can be found by **backwards induction**

start at each of the “last” decision nodes : nodes immediately preceding terminal nodes

at these “last” nodes, the Nash equilibrium to the sub–game is simply whatever action is best for the player making the move

so find that best action

now label that node with the payoff from the action which will be chosen (and with the name of the action chosen)

we’ve just reduced the game to a shorter extensive–form game now move up one level, to the last decision node of this new game, and do the same thing

keep doing this, and eventually you get to the top

the actions chosen at each node constitue a set of SPNE strategies, and the payoffs are the payoffs which will result if the game ever gets to that node

(this corresponds to solving the strategic form by interated elimination of weakly dominated strategies)