Coalitions and Blocking

"barter" exchange economy

now people **own** the endowments

so that

$$\mathbf{e} \equiv \sum_{i=1}^{I} \mathbf{e}^i$$

$\mathbf{e}^i$ is person i's endowment vector

people could form coalitions to exchange their endowments

if $S \subset \{1, 2, 3, \ldots, I\}$ is a coalition, what allocations can the members of the coalition get?

the allocation which gives $\mathbf{y}^i$ to person i (with i in the coalition S) is feasible for the coalition if

$$\sum_{i \in S} y_k^i \leq \sum_{i \in S} e_k^i \tag{1}$$

for each good k

now consider some allocation $\mathbf{x}$ for the **whole** economy — not just the coalition S

definition : the coalition S is said to "**block**" the allocation $\mathbf{x}$ with allocation $\mathbf{y}$ if

i $\mathbf{y}$ is feasible for the coalition : that is $\mathbf{y}$ obeys condition (1)

ii $u^i(\mathbf{y}^i) \geq u^i(\mathbf{x}^i)$ for each person $i \in S$

iii $u^h(\mathbf{y}^h) > u^h(\mathbf{x}^h)$ for some person $h \in S$

The Core

definition : a feasible allocation $\mathbf{x}$ is in the **core**, if there is **no** coalition $S \subseteq \{1, 2, 3, \ldots, I\}$ of people which can block $\mathbf{x}$

so $\mathbf{x}$ is in the core if there is no way that a group of people can go off on their own, and all do better than they would under $\mathbf{x}$

if $\mathbf{x}$ is in the core, then $\mathbf{x}$ must be "individually rational" : $u^i(\mathbf{x}^i) \geq u^i(\mathbf{e}^i)$ for each person i

if $\mathbf{x}$ is in the core, then $\mathbf{x}$ must be Pareto efficient

why? because if $\mathbf{x}$ were not Pareto optimal, then there would be some other feasible allocation $\mathbf{y}$ which was Pareto–preferred ; but then a subset S consisting of **everyone** could block $\mathbf{x}$ with $\mathbf{y}$

with 2 people, that is exactly what the core is : the set of individually rational, Pareto efficient allocations (that is, the part of the contract curve which is between the indifference curves of the two people in the Edgeworth Box)

BUT .. if $I > 2$, the core is **smaller** than that

if $I > 2$, there are allocations which are Pareto efficient, and individually rational, but which are **not** in the core

example : $I = 4$

$U^i(\mathbf{x}^i) = x_1^i x_2^i$ for each of the 4 people

$\mathbf{e}^1 = \mathbf{e}^2 = (2, 0)$ and $\mathbf{e}^3 = \mathbf{e}^4 = (0, 2)$

since $u^i(\mathbf{e}^i) = 0$ for each person, then any allocation in which $x_j^i > 0$ for all i, j is individually rational

any allocation $\mathbf{x}$ in which $x_1^i = x_2^i$ for all people i is Pareto efficient so $\mathbf{x}^1 = \mathbf{x}^2 = (0.4, 0.4)$, and $\mathbf{x}^3 = \mathbf{x}^4 = (1.6, 1.6)$ is Pareto efficient and individually rational

this allocation yields $u^1 = u^2 = 0.16$, $u^3 = u^4 = 2.56$

it is **not** in the core

$S = \{1, 2, 3\}$ can block $\mathbf{x}$ with :

$\mathbf{y^1} = \mathbf{y}^2 = (1.2, 0.2)$, $\mathbf{y}^3 = (1.6, 1.6)$

$$y_1^1 + y_1^2 + y_1^3 = 1.2 + 1.2 + 1.6 = 4 = e_1^1 + e_1^2 + e_1^3$$

$$y_2^1 + y_2^2 + y_2^3 = 0.2 + 0.2 + 1.6 = 2 = e_2^1 + e_2^2 + e_2^3$$

and

$u^1 = u^2 = 0.24$, $u^3 = 2.56$