Signalling

the model

"nature" chooses workers' ability a

each worker knows her own ability, but firms do not observe nature's choice of a

workers move first, choosing an education level e

[terminology : when the informed agents (workers) move first, it's a "signalling" model ; when the uninformed agents (firms) move first, it's a "screening" model) ; cf. *Jehle and Reny* 8.1.2 and 8.1.3]

firms then offer wages, which can depend on workers' education levels

but not on ability

since firms can observe e, but not a

output per worker : a

so (in this model) education does **not** add to a worker's productivity

cost of education — to a worker of ability a

$$\frac{e}{a}$$

key feature : education is more costly to less able workers

firms' strategies : wages which they pay

many firms : so that competition will lead to zero profits

Perfect Information

if firms could observe each worker's a, and condition wages on a (as well as e)

then in equilibrium,

$$W(e, a) = a \tag{1}$$

competition drives wage up to value of worker's output

wage does not depend on e, since education does not add to productivity

no workers choose any education : it's costly, and does not increase her wage

this outcome is efficient

No Information

if **neither** workers nor firms could observe ability, then we would have an equilibrium in which the wage was $\bar{a}$, the average ability in the population

wages would not rise with education, workers would not acquire any education, the outcome would be efficient

point : asymmetry of information matters

Back to the Model

firms' strategies are wage functions $W(e)$ which may depend on a workers' education levels

equilibrium :

workers : choose an education level $E(a)$ to maximize net payoff

$$\max_e W(e) - \frac{e}{a} \tag{2}$$

solution to (2) defines workers' education choice function $E(a)$

$$E(a)$$

first–order condition

$$W'(e) = \frac{1}{a} \tag{3}$$

if $W(e)$ is concave, equation (3) implies that $E(a)$ is an increasing function : more able workers choose to acquire more education

and the solution to (3) will be optimal for the worker only if $W(e)$ is concave

Firms' Beliefs

firms maximize expected profits : expected value of output, minus wages

so the expected profit from offering a wage $W(e)$ to a worker of education level e is

$$A(e) - W(e) \tag{4}$$

where $A(e)$ is the firm's belief about the **average** ability level of all workers who have chosen to acquire education level e

for an equilibrium, we need

i correct beliefs : $A(e)$ is the actual expected productivity of workers of education level e [calculated using Bayes's Rule]

ii zero profits : $A(e) = W(e)$ for any education level e which is actually chosen by some workers

(competition drives profits to zero)

A Separating Equilibrium

$$W(e) = \sqrt{2e} \tag{5}$$

$$E(a) = \frac{a^2}{2} \tag{6}$$

firms?

from (6) each different abiliy level a of worker chooses a different level of education

so the only $A(e)$ which is consistent with Bayes's Rule is what we get from inverting (6) :

$$A(e) = \sqrt{2e} \tag{7}$$

equations (5) and (7) imply that firms make zero profit on each type of worker

workers

maximize $\sqrt{2e} - \frac{e}{a}$ with respect to e

solution to f.o.c. :

$$\frac{\sqrt{2}}{2\sqrt{e}} = \frac{1}{a} \tag{8}$$

or

$$E(a) = \frac{a^2}{2} \tag{9}$$

which is exactly equation (6)

so equations (5), (6) and (7) define a sequential equilibrium

Properties of the Separating Equilibrium

workers choose to acquire education, even though it is not productive

why? because firms reward education with higher wages

why do firms pay more for educated workers?

because they believe that they are more productive

and in equilibrium these beliefs are correct

since more able workers find it less costly to acquire education