Cournot Oligopoly

quantities are strategic variables

meaning : each firm chooses a quantity q^i to produce

each firm's price is then determined by the market demand

"standard" case : homogeneous output

products of each firm in the industry are **perfect substitutes** for each other

then price is determined as $p(Q)$, where $p(\cdot)$ is the industry's aggregate inverse demand function, and where

$$Q \equiv q^1 + q^2 + \cdots + q^J$$

is industry output

profit of firm i

$$\pi^i = p(Q)q^i - C(q^i) \tag{1}$$

depends on other q^j's through the effect of aggregate output Q on price

since $p'(Q) < 0$, π^i decreases with each other q^j

profit maximization by firm i : choose q^i to maximize π_i, taking each other q^j as given

first–order condition

$$\frac{\partial \pi_i}{\partial q^i} = p(Q) - MC(q^i) + p'(Q)q^i = 0 \tag{2}$$

example : linear demand ; constant MC

assume that

$$p = a - bQ \qquad a > 0, b > 0 \tag{3}$$

and

$$C(q^i) = cq^i \qquad a > c > 0 \tag{4}$$

then the first–order condition (2) becomes

$$\frac{\partial \pi^i}{\partial q^i} = a - b(\sum_{j=1}^{J} q^j) - c - bq^i = 0 \tag{5}$$

or

$$2bq^i = a - c - bQ_{-i} \tag{6}$$

reaction functions

where

$$Q_{-i} \equiv \sum_{j \neq i} q^j = Q - q^i$$

equation (6), which can also be written

$$q^i = \frac{a-c}{2b} - \frac{Q_{-i}}{2} \qquad (7)$$

is the **reaction function** for firm i

slope of reaction function : from equation (7),

$$\frac{\partial q^i}{\partial q^j} = -\frac{1}{2} \qquad i \neq j \qquad (8)$$

more generally...

for general demand, cost functions, equation (2) implies that

$$\frac{\partial q^i}{\partial q^j} = -\frac{p'(Q) + p''(Q)q^i}{2p'(Q) + p''(Q)q^i - C''(q^i)} \qquad (9)$$

$$0 \geq \frac{\partial q^i}{\partial q^j} > -1$$

if $C'' \geq 0$ and if $p'' \geq 0$

equilibrium : linear case

equilibrium : each firm's q^i maximizes its own profits, given other firms' output choices q^j

symmetric equilibrium : $q^1 = q^2 = \cdots = q^J \equiv q$

means that $Q_{-i} = (J-1)q$

so that (6) becomes

$$2bq = a - c - b(J-1)q \tag{10}$$

or

$$q = \frac{a-c}{(J+1)b} \tag{11}$$

implying

$$Q = \frac{J}{J+1}\frac{a-c}{b} \tag{12}$$

and

$$p = \frac{a}{(J+1)} + \frac{Jc}{(J+1)} \tag{13}$$

or

$$p - c = \frac{a-c}{J+1} \tag{14}$$

which means

$$\pi^i = (p-c)q^i = \frac{(a-c)^2}{(J+1)^2 b} \tag{15}$$

so that total industry profits are

$$\Pi = \frac{J}{J+1}\frac{(a-c)^2}{(J+1)b} \tag{16}$$

$$\frac{\partial \Pi}{\partial J} < 0 \tag{17}$$