Bertrand Duopoly

prices are the strategic variables

quantity sold by firm 1 : $q^1(p^1, p^2)$

$$\pi^1 = p^1 q^1(p^1, p^2) - C[\mathbf{w}, q^1(p^1, p^2)] \tag{1}$$

prices chosen simultaneously

(Nash) equilibrium : a pair of prices (p^1, p^2), such that p^1 maximizes π^1 ,given p^2, and such that p^2 maximizes π^2, given p^1

“benchmark” case

i **homogeneous** output ; i.e. firm 1’s product is a **perfect substitute** for firm 2’s

ii constant returns to scale : $C(\mathbf{w}, q) \equiv cq$, where c is some constant (which depends on input prices)

market demand : $D(p)$ is the equation of the market demand curve for the homogeneous product

homogeneous product $\rightarrow$ buyers always buy from cheapest source

implication

demand for firm i's product

if $p^1 > p^2$, then $q^1(p^1, p^2) = 0$

why? everyone buys from (cheaper) firm #2

if $p^1 < p^2$, then $q^1(p^1, p^2) = D(p^1)$

everyone buys from firm #1

if $p^1 = p^2$, then

$$q^1(p^1, p^2) = q^2(p^1, p^2) = \frac{1}{2} D(p^1) \tag{2}$$

(rule (2) is not essential)

Nash equilibrium

$p^1 > p^2 > c$?

can't be an equilibrium : firm #1 makes zero profits (since it has zero sales) ; given p^2, firm #1 can do better than that, by choosing some p' between c and p^2 (if $c < p' < p^2$, then firm #1 will get positive sales from charging the price p', and will make positive profits, since $p' > c$)

similarly, $p^2 > p^1 > c$ cannot be a Nash equilibrium

what about $p^1 = p^2 > c$?

can't be an equilibrium

when $p^1 = p^2 > c$, firm 1's profits are

$$\frac{1}{2}[p^2 - c]D(p^2)$$

by lowering it price very slightly, from p^2 to $p' = p^2 - \epsilon$, firm #1 lowers its profit margin very slightly, from $p^2 - c$ to $p' - c$

but this slight price reduction will more than **double** its sales : from $\frac{1}{2}D(p^2)$ to $D(p') > D(p^2)$

if ϵ is small enough (p' close enough to p^2), this change in strategy must increase firm 1's profits, so that $p^1 = p^2 > c$ cannot be a Nash equilibrium

what's left?

how about $p^1 > p^2 = c$?

also can't be a Nash equilibrium : firm #2 gets all the sales, but has zero profits (since its price equals its average cost) ; given p^1, firm #2 can increase profits by raising its price from $p^2 = c$ to some p' with $p^1 > p' > c$; if $p' < p^1$ firm #2 will still get all the sales, but if $p' > c$ firm #2 will now make a positive profit per unit sold

clearly there can be no Nash equilibrium in which **either** firm charged a price below cost : the lower–price firm will make negative profits ; it always could do better by charging some price above c, which guarantees profits are 0 or positive the unique Nash equilibrium in this market is $p^1 = p^2 = c$

if $p^2 = c$, firm 1 makes zero profits by charging a price of $p^1 = c$; but it cannot do better than that : any price above c gets it zero sales, and any price below c gives it negative profits

very different results than Cournot : with homogeneous output, and constant costs, a little competition is the same as perfect competition as long as the number of firms J in the market is greater than 1, then the equilibrium price will be c, whether J is 2, or 3, or 1000