Conditions for Asymmetric Equilibrium

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Abstract

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1 Preliminaries

When modelling fiscal competition, it is often assumed that jurisdictions are identical in tastes and technology, but differ in a single exogenous characteristic, such as population, or endowment of some factor.

A question often addressed in these models is the effect of the asymmetry on the equilibrium, when jurisdictions’ governments behave non–competitively. For example, “do larger countries levy higher source–based tax rates on capital?”, “do countries with higher capital ownership set higher tariff rates?”.

When it is assumed that the jurisdictions have identical tastes and technologies, it usually follows that the outcome will be symmetric if the two jurisdictions’ governments choose the same value for the variable they are choosing. That is, if the two countries levied the same tax rate on capital income, the resulting endogenous (per capita) capital demand would be the same in each country. Or if each country set the same tariff rate, then there would be no trade, and countries would have identical production patterns.

When endogenous variables are the same, then it may be easy to compare the payoff functions of the country’s decision makers. On the other hand, if countries differ in their factor usage, or in the composition of residents’ consumption bundles, then such a comparison may be more difficult. These payoff functions, or their derivatives, depend on the marginal return to inputs, or on residents’ marginal willingness to pay for some consumption good.

To be more concrete, consider a canonical tax competition model, such as many of the models analyzed in Wilson (1999), or in Keen and Konrad (2013). If output per capita in country $i$ is some function $f(k_i)$ of the capital–labor ratio $k_i$ in the country, then the net–of–tax income of a resident of country $i$ can be written

$$x_i = f(k_i) - f'(k_i)k_i + f'(k_i)e_i - t_ie_i$$  \hspace{1cm} (1)
when each resident has an endowment $e_i$ of mobile capital, when output per person in the country is $f(k_i)$, and where each resident gets an equal share of the country’s production, after capital owners have been paid a return equal to the marginal product of capital. Here $t_i$ is the unit tax rate on capital employed in the country, the strategic variable chosen by the country’s government. Implicit in equation (1) is the assumption of perfect mobility of capital, so that the net return to capital, wherever it is employed, is $f'(k_i) - t_i$. Differentiation of (1) implies that

$$\frac{\partial x_i}{\partial t_i} = \left[f''(k_i)(e_i - k_i)\right] \frac{\partial k_i}{\partial t_i} - e_i$$

(2)

Now if two countries have identical technologies, and if $k_1 = k_2$, then it is straightforward to show that

(i) $\frac{\partial x_1}{\partial t_1} > \frac{\partial x_2}{\partial t_2}$ if countries differ only in size, and if country 1 is larger than country 2

(ii) $\frac{\partial x_1}{\partial t_1} > \frac{\partial x_2}{\partial t_2}$ if countries differ only in per–capita endowments, and $e_1 < e_2$.

But if $k_1 \neq k_2$, then a comparison of the values of expression (2) depends on how the expression $f''(k_i)$ varies with $k_i$. To make the comparison relatively simple, one needs to assume a quadratic production technology.

Wilson (1991, section 4) shows how such an assumption is not needed, if the reaction functions of the two countries are well–behaved. In the particular model just described, continuity of countries’ reaction functions is sufficient for properties of reaction functions on the diagonal to characterize the nature of the Nash equilibrium.

The purpose of this note is to generalize Wilson’s argument.

**Problem :** There are two decision makers, 1 and 2, choosing strategic variables $t_1$ and $t_2$ non–cooperatively. The payoff to decision maker $i$ is $\pi_i(t_1, t_2)$. We have information about the relative values of $\frac{\partial \pi_i}{\partial t_i}$ only when $t_1 = t_2$. What does this information imply for the Nash equilibrium values of $t_1$ and $t_2$?

**Solution :** If reaction correspondences of the two decision makers are well–behaved, and if $\frac{\partial \pi_1}{\partial t_1} > \frac{\partial \pi_2}{\partial t_2}$ whenever $t_1 = t_2$, then there must exist a Nash equilibrium in which $t_1 > t_2$.

In other words, sufficient conditions are provided for information about pay–offs when strategic variables are equal in value to tell us which player chooses the higher value for that strategic variable in equilibrium.
2 The Result

There are two players, who choose actions $t_1$ and $t_2$ non-cooperatively, so as to maximize $\pi^1(t_1, t_2)$ and $\pi^2(t_1, t_2)$ respectively. The actions $t_1$ and $t_2$ are non-negative numbers. There is also some finite upper bound $T$ on the best responses. In particular

1. There is some $0 < T < \infty$ such that

$$\frac{\partial \pi^1}{\partial t_1}(T, t_2) < 0 \quad \text{for all} \quad 0 \leq t_2 \leq T$$

and

$$\frac{\partial \pi^2}{\partial t_2}(t_2, T) < 0 \quad \text{for all} \quad 0 \leq t_1 \leq T$$

and

$$\frac{\partial \pi^1}{\partial t_1}(0, t_2) > 0 \quad \text{for all} \quad 0 \leq t_2 \leq T$$

and

$$\frac{\partial \pi^2}{\partial t_2}(t_1, 0) > 0 \quad \text{for all} \quad 0 \leq t_1 \leq T$$

2. For every $0 \leq t_2 \leq T$ ($0 \leq t_1 \leq T$) player 1 (player 2) has a unique best response $t^R_1(t_2)$ ($t^R_2(t_1)$) which maximizes $\pi^1(t, t_2)$ ($\pi^2(t_1, t)$) over $[0, T]$. This best response function $t^R_1(t_2)$ ($t^R_2(t_1)$) is a continuous function of $t_2$ ($t_1$).

3. $\frac{\partial \pi^1}{\partial t_1}(t, t)$ and $\frac{\partial \pi^2}{\partial t_2}(t, t)$ are both decreasing functions of $t$.

**Proposition 1** Under assumptions 1–3, if

$$\frac{\partial \pi^1}{\partial t_1}(t, t) > \frac{\partial \pi^2}{\partial t_2}(t, t) \quad \text{for all} \quad 0 \leq t \leq T,$$

then there exists a Nash equilibrium pair of actions $(t^N_1, t^N_2)$ with $t^N_1 > t^N_2$.

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1 So the results here would not be applicable to “attachment to home” models, such as Kanbur and Keen (1993), in which there are discontinuities in one of the reaction correspondences.
Proof Assumptions 1 and 3 ensure that, for each player, there is a unique point \((t^D_i, t^D_i)\) on the diagonal \(t_1 = t_2\) such that \((t^D_i, t^D_i)\) satisfies the first–order conditions for maximization of \(\pi^i\).

Assumptions 2 and 3 imply that \((t^D_i, t^D_i)\) is actually on the reaction curve of player \(i\), since that curve must cross the diagonal, and points on the reaction curve must satisfy the first–order conditions for maximization of \(\pi^i\).

In a graph in \([0, T] \times [0, T]\], put \(t_1\) on the horizontal axis and \(t_2\) on the vertical. Label \((t^D_1, t^D_1)\) and \((t^D_2, t^D_2)\) as points \(A\) and \(a\) respectively. The hypothesis of the Proposition implies that \(A\) must be above and to the right of \(a\).

Player 2’s best reaction \(t^B_2(T)\) must be some point \(b\) on the right edge of the graph, from assumptions 1 and 2. Player 1’s best reaction \(t^B_1(0)\) must be some point \(B\) on the bottom edge of the diagram, again from assumptions 1 and 2.

From assumption 2 there must be a continuous curve connecting points \(A\) and \(B\) which is part of player 1’s reaction curve, and there must be a continuous curve connecting points \(a\) and \(b\) which is part of player 2’s reaction curve. Assumption 3 ensures that neither of these curves can cross the diagonal (except at \(A\) and \(a\) respectively).

So the 2 curves must intersect, somewhere below and to the right of the diagonal. This intersection must be a Nash equilibrium.

Of course assumption 2 is the strong one. In the introduction, I mentioned the role of strong assumptions about higher derivatives of production functions, in order to sign expression (2). By requiring that reaction correspondences are well–behaved, those assumptions have, in a sense, just been made implicit.

But, typically, similarly strong assumptions must be made in this sort of model, simply to guarantee that a (pure strategy) Nash equilibrium exists. In some sense, Proposition 1 says : “if we are confident that a Nash equilibrium exists, then behaviour of players along the diagonal is sufficient to determine the qualitative properties of that equilibrium”.

References


