abstract: If there are scale economies associated with local public infrastructure, then it may be efficient to concentrate public investment in a few large projects. If this is the case, then one of the costs imposed by log-rolling in legislatures may be that distributive spending is excessively dispersed among regions. The larger the coalition doing the log-rolling, the worse this problem. There may be a trade-off in formulating the optimal size of winning coalition: increasing the size lessens common pool problems but exacerbates this dispersion problem. Here the sum of net benefits from public investment, added across all regions, is calculated as a function of the size of the winning coalition. The calculation is then extended to a legislature in which the group proposing legislation is a distinct subset of the winning coalition.

* See Battistella and Dunleavy (1975) for an account of Ms. Foxe’s role in the political career of Representative Wilbur Mills (D,Ark.), and her contribution to the decline of the seniority system in the United States Congress.
As Persson and Tabellini (2000) note, “(m)any economic policy decisions create concentrated benefits for a few well-defined groups, with the cost diffused in society at large”. Often, these groups are defined by location. The tendency for legislatures to spend too much on “pork barrel” projects is usually attributed to this concentration of benefits and diffusion of costs. Persson and Tabellini (2000) refer to this over-spending as a common-pool problem: the coalition which decides spending will not internalize the costs imposed on non-members.

Typically, analyses of distributive spending assume that the net benefits to residents of some district are some (increasing, concave) function of the amount of distributive spending in the district itself, minus the taxes paid by the district’s residents on projects in all districts.

Suppose that spending is somehow decided by a coalition of representatives in some legislature. The model just described implies that larger coalitions are more efficient, since a larger fraction of the cost of projects is internalized. Shepsle and Weingast (1981) use this model to demonstrate the superiority of a universalist norm for a legislature, in which projects are decided by a coalition of the whole, to a majoritarian norm, in which a minimum winning coalition makes the decisions.

In fact, in the simplest possible setting, a legislature with a universalist norm will yield an efficient outcome, if one’s criterion is the simple sum of net benefits across all districts. This model, for example, has been used by Besley and Coate (forthcoming) to show the superiority of ‘centralized’ states to “decentralized”, if the centralized state uses a legislature with a universalist norm to decide expenditure. (Besley and Coate extend the model by allowing for externalities among districts, which further strengthens the case for centralization. They also consider strategic delegation by voters, an issue which will not be addressed here)

However, the only way to reward coalition members in this setting is by undertaking public expenditure in their districts. That is, a maintained hypothesis throughout the literature on distributive spending is that payments in cash cannot be made to individual legislators (or to constituents of a specific legislator). Given the importance of broad-based taxes, this hypothesis seems to be a reasonable one. It follows from this hypothesis that public spending will be diffused among more districts, the broader the coalition of legislators which makes the spending decisions.

With increasing returns, this diffusion is a cost to having larger coalitions. In this paper, a very simple model of legislative decision making is used to examine the efficiency of different coalition sizes. Throughout, I will use a single measure of efficiency, the sum of net benefits in all districts. This measure can be justified as an ex ante measure of welfare, behind the usual veil of ignorance. Suppose that a legislature contains \(N\) identical districts, and that policies in that legislature are chosen by a winning coalition of \(M \leq N\) legislators. Then if each legislator has an ex ante probability of \(M/N\) of being in the winning coalition, the sum of net benefits will be proportional to each district’s expected net benefits.

A large \(M\) means more of the costs are internalized. But the increasing returns mean fewer scale economies if spending is divided finely among districts. The sort of scale economies which I have in mind are agglomeration economies which are concentrated in a relatively small area. In particular, the net benefit function for each district, presented below, can be derived from the
following production structure. Some factor, such as skilled labour, is perfectly mobile among districts. Public expenditure in a district is used as an input to an industry, which exhibits economies of scale at the industry level, but not at the firm level. Thus districts which have a high level of public expenditure will attract more of the mobile input, increasing the agglomeration economies. This underlying model is presented in more detail in Bucovetsky (2003), which analyzes the distribution of public expenditure in a decentralized state, in which each district makes its own expenditure choice, using its own tax revenues. In this model, there is a well-defined optimum, using the simple sum-of-net-benefits criterion for efficiency. Public expenditure should be concentrated in a single district. Under centralization, this concentration can be achieved only if legislative decisions are dictated by a single member. But a dictator would over-spend, since a fraction \((N-1)/N\) of costs of spending in her district are paid by residents of other districts. The only way of reducing this common-pool externality is to have a decisive coalition which is larger than 1. But adding members into a coalition requires spreading public spending among the districts, since I assume that here is no other way of compensating coalition members.

The first few sections of the paper examine this very simple trade-off. In particular, no attention is paid to the structure of the legislature. It is just assumed that the legislative norm is to let a coalition of size \(M\) dictate policy. In contrast, most of the literature on distributive politics examines the process of voting much more carefully. In papers such as Baron and Ferejohn (1989), and Ferejohn, Fiorina and McKelvey (1987) voting is analyzed as a game in extensive form, in which a “proposer” for legislation must obtain support from some fraction of the legislature. Lockwood (2002) has applied this sort of model to the sort of efficiency issue examined here, and by Besley and Coate (forthcoming).

In sections 4–6 of the paper, I consider a somewhat less simple legislature. The model is still very simple. The main extension introduced is a distinction between “proposers” and “approvers”. In these subsequent sections, the ruling coalition no longer has dictatorial power. Instead they have a monopoly on proposing legislation. But the legislation they propose must be approved. This sort of model is closer to those analyzed in the political literature, and is perhaps closer to behaviour in the U.S. Congress, where committees do have considerable power over what legislation gets onto the floor.

So the strict seniority system seemed to give enormous power to committee chairs in the U.S. Did the benefits of concentration implied by agglomeration economies justify such a system on efficiency grounds? The simple model presented in sections 1–3 below provides a negative answer. However, if committee chairs could only get legislation approved by adding projects in other legislators’ districts, the answer may change. It may be more efficient to have a committee chair write legislation alone, than to have it set by a group of legislators.

1. The Model

A country is divided into \(N\) districts. Each district is identical. The country contains \(\bar{L}\) skilled
workers, each of whom is perfectly mobile among districts. These skilled workers are employed only in the technology sector. Total output in the technology sector in district $i$ is

$$Q_i = B L_i^a G_i^g$$  \hspace{1cm} (1)$$

where $L_i$ is employment of skilled labour in the district, and $G_i$ is the size of the public investment in the district. It will be assumed that both $a$ and $g$ are between 0 and 1 in value.  

The different districts’ outputs from the technology sector may differ from each other. The value of total national output from the technology sector is

$$Y = \left( \sum_{i=1}^{N} Q_i^\rho \right)^{1/\rho} \hspace{0.5cm} 0 < \rho \leq 1$$  \hspace{1cm} (2)$$

( so that $\rho < 1$ implies that projects in different districts are imperfect substitutes for each other ).

Skilled workers are paid the value of their marginal product. Since they are perfectly mobile, this wage must be equal in all districts in which any skilled labour is employed. This mobility then implies an allocation of labour

$$L_i = \frac{(G_i^\gamma \sum_{j=1}^{N} G_j^\gamma \bar{L})}{\sum_{j=1}^{N} G_j^\gamma}$$  \hspace{1cm} (3)$$

where

$$\gamma \equiv \frac{\rho g}{1 - \rho a}$$

and a value of technology output in district $i$ of

$$V_i = A G_i^\gamma \left[ \sum_j G_j^\gamma \right]^{g/\gamma - 1}$$  \hspace{1cm} (4)$$

where $A = B \bar{L}^a$.

The legislator for each district cares about the value $V_i$ of output in her district, minus any taxes paid by residents of the district. As mentioned in the introduction, scale economies are what cause diversification to be inefficient. The assumption that there are scale economies is

**ASSUMPTION A1**: $a + g > \frac{1}{\rho}$

Assumption A1 is equivalent to assuming that $\gamma > 1$. It requires that government expenditure be quite productive ( high $g$ ), and that the different regions’ projects be fairly close substitutes ( high $\rho$ ).

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1 This is the same underlying model as the one used in Bucovetsky (2003), which provides a little more detail on the derivations.

2 The parameter $a$ is the overall elasticity of aggregate technology output $Q_i$ in the district with respect to total skilled labour employment $L_i$. In particular, if there are economies of scale which are internal to the industry but external to the firm, $a$ could be considerably larger than the coefficient of an individual firm’s own employment of skilled labour in its own Cobb–Douglas production function. That means that $a$ could be much larger than the share of skilled labour in the technology sector’s costs, if factors are paid the value of their marginal product to the firms.
Units are chosen so that $G_i$ is also the total cost of government expenditure in district $i$. Given the fixed supply of skilled labour, there is a simple measure here of the net value of output, namely the value of output $Y$, minus the cost of public investment. From equations (1), (2) and (3), this net value is

$$W = A\left[\sum_{i=1}^{N} G_i^{\gamma} - \sum_{i=1}^{N} G_i \right]$$

which is maximized by concentration of all public investment in a single district if (and only if) $\gamma > 1$, and then by choosing a level of investment

$$G^* = Ag^{1/(1-g)}$$

in the one district.

My measure of efficiency, when public expenditure is decided by the legislature, is just $W$.

2. The Winning Coalition’s Choice

I assume further that

ASSUMPTION A2 : Each member of the winning coalition gets the same payoff.

This assumption can be motivated by having the coalition members bargain behind closed doors, before proposing legislation — with a legislature which uses a closed rule. All districts have been assumed identical ex ante. Therefore, virtually any axiomatic bargaining rule would predict an equal division of the surplus among coalition members.

A coalition chooses a pattern of public expenditure. It is assumed that the cost of any projects is divided equally by all $N$ districts, so that the payoff to district $i$ is

$$V_i - \frac{1}{N} \left( \sum_{j=1}^{N} G_j \right)$$

From equation (4), there is no reason for members of the coalition to do any public investment in any non–member’s district : this investment is costly, and lowers $V_i$ of each of the members. Assumption A2 then implies that the coalition chooses a common public investment level $G$ of members, so as to maximize the net benefits

$$\pi_i^m = AG^\gamma [MG^\gamma]^{g/\gamma - 1} - \frac{M}{N} G = AG^g M^{g/\gamma - 1} - \frac{M}{N} G$$

of each member of the coalition.

Maximization of expression (7) then yields the optimal level of public investment in each member district of the coalition,

$$G^w(M) = [gAN]^{1/(1-g)} M^{-\beta}$$

5
where

\[ \beta \equiv \frac{2\gamma - g}{\gamma(1 - g)} > 0 \]

so that, not surprisingly, increasing the size of the coalition will reduce public investment in each member’s district.

It also follows that total public investment implemented by the coalition is

\[ MG^w(M) = \left[gAN\right]^{1/(1-g)}M^{1-\beta} \quad (9) \]

From the definition above, the following lemma applies.

**LEMMA 1:** Increasing the size \(M\) of the coalition decreases total public investment \(MG^w(M)\).

**PROOF:** From equation (8) the lemma holds if (and only if) \(\beta > 1\). That inequality will hold if

\[ 2\gamma - g - \gamma(1 - g) > 0 \quad (9) \]

The left side of inequality (9) is positive when \(\gamma = 1\), and is increasing in \(\gamma\), proving the lemma. •

**LEMMA 2:** The total amount of public investment \(MG^w(M)\) is larger than the efficient amount \(G^*\) if the coalition size is small, but smaller than the efficient amount if the coalition is large.

**PROOF:** When \(M = 1\), then equation (8) implies that

\[ G^w(1) = [Ag]^{1/(1-g)}N^{1/(1-g)} \]

which, not surprisingly, exceeds \(G^*\) when \(N > 1\). Equation (8) may also be written

\[ G^w(M) = [Ag]^{1/(1-g)}\left(\frac{N}{M}\right)^{1/(1-g)}M^{-\delta} \quad (10) \]

where

\[ \delta \equiv \frac{\gamma - g}{\gamma} \frac{1}{1 - g} \]

implying that

\[ NG^w(N) = G^*M^{1-\delta} \]

so that universalist norms imply too little aggregate public expenditure if (and only if) \(\delta > 1\). But \(\delta > 1\) if and only if

\[ \gamma - g - \gamma(1 - g) > 0 \quad (11) \]

The left side of inequality (11) equals 0 when \(\gamma = 1\), and is increasing in \(\gamma\), proving the lemma. •
Lemmas 1 and 2 suggest that norms can be too large when there are increasing returns to scale. If $M$ is close enough to $N$, then lemma 2 shows that aggregate investment will be too small. From lemma 1, further increases in $M$ both exacerbate the aggregate under-provision, and diversify it even more inefficiently.

3. Social Welfare

If I denote by $W(M)$ the net aggregate social welfare, when a coalition of size $M$ decides public investment, then

$$W(M) = A[G^w(M)]^gM^{g/\gamma} - MG$$

which also can be written

$$W(M) = M\pi^M - \frac{N - M}{N}MG$$

where $\pi^M$ is the payoff to a member of the winning coalition: expression (12) is just the sum of the winners’ payoffs, minus the costs to non-members.

Differentiating (12) with respect to $M$,

$$W'(M) = \frac{\partial}{\partial M}[M\pi^M] - G\frac{N - 2M}{N} - M\frac{N - M}{N}\frac{\partial G}{\partial M}$$

Using the first-order condition for the coalition’s maximization,

$$gAM^{-(\gamma-g)/\gamma}G^{\gamma-1} = \frac{M}{N}$$

the first term on the right side of (13) is

$$\frac{\partial}{\partial M}[M\pi^M] = -G\left(\frac{M}{N}\right)[2 - \frac{1}{\gamma}]$$

From equation (8),

$$\frac{\partial G^w(M)}{\partial M} = -\frac{2\gamma - g}{\gamma(1-g)}\frac{G}{M}$$

Substituting (15) and (16) into (13) yields

$$W'(M) = \frac{G}{N}[M - (N - M)][\frac{2\gamma - g}{\gamma} - \frac{1}{1-g}] - (1 - \frac{1}{\gamma})M$$

LEMMA 3 : Social welfare is a single-peaked function of the winning coalition size $M$,

PROOF : Equation (17) can be re-written

$$W'(M) = \frac{G}{N}[\frac{2\gamma - g}{\gamma} - \frac{1}{1-g}]N - \frac{2\gamma - 1}{\gamma(1-g)}M$$
The coefficient of $M$ in equation (18) must be negative, since $\gamma > 1 > g$.

**PROPOSITION 1**: The most efficient size of winning coalition is either the smallest integer greater than $M^*$, or the largest integer less than $M^*$, where
\[
M^* = \frac{\gamma + (\gamma - 1)g}{2\gamma - 1} N
\] (19)

**PROOF**: From equation (18), $W(M)$ reaches a maximum at $M = M^*$.

**LEMMA 4**: The ideal coalition size $M^*$ is a decreasing function of the scale parameter $\gamma$, and an increasing function of the public investment parameter $g$.

**PROOF**: Differentiation of equation (19) yields
\[
\frac{\partial M^*}{\partial \gamma} = -(2\gamma - 1)^{-2}(1 - g) < 0 \quad (20)
\]
and
\[
\frac{\partial M^*}{\partial g} = \frac{\gamma - 1}{2\gamma - 1} N > 0 \quad (21)
\]

**PROPOSITION 2**: If the number $N$ of districts is larger than $2/g$, then the most efficient size of winning coalition is greater than $N/2$.

**PROOF**: As $\gamma \rightarrow \infty$, equation (19) shows that $M^* \rightarrow \frac{(1+g)}{2} N$.

Since Lemma 2 shows that $M^*$ is a decreasing function of $\gamma$, then $\frac{M^*}{N} < \frac{1+g}{2}$.

Of course, $M^*$ need not be an integer. The largest integer less than or equal to $M^*$ must be greater than or equal to
\[
\frac{1 + g}{2} N - 1
\]
which must exceed $N/2$ if $N > 2/g$.

As long as there are some scale economies, then universalism is not the best possible norm.

**PROPOSITION 3**: If $N > \frac{2\gamma - 1}{(\gamma - 1)(1-g)}$, then the ideal coalition size is $N - 1$ or smaller.

**PROOF**: Equation (19) shows that $M^* = N$ whenever $\gamma = 1$, so that (from Lemma 4), $M^* < N$ whenever $\gamma > 1$.

Given that the ideal coalition size must be an integer, it will be strictly less than unanimity whenever $M^* < N - 1$. From equation (19), this will be true whenever
\[
N > \frac{2\gamma - 1}{(\gamma - 1)(1-g)}
\]
Clearly, in this model it is not ideal to have a very small clique having completely power over distributive expenditure. However, an even stronger statement can be made. Substitution of the coalition’s first–order condition (14) into the definition (11) of overall social welfare yields

\[ W(M) = MG^w(M)\left[\frac{M}{Ng} - 1\right] \tag{22} \]

Remember, the benchmark here — the allocation for which social welfare equals zero — is an allocation in which no district gets any distributive public expenditure at all. (That is, \( W(M) \) is measured relative to a world in which all \( G_i \)'s are zero, and in which no taxes need to be levied.)

Then

**PROPOSITION 4**: A coalition of size smaller than \( gN \) leads to a worse overall outcome than having no public expenditure at all.

**PROOF**: Equation (22) shows \( W(M) < 0 \) whenever \( M/Ng < 1 \).

**COROLLARY**: \( M = 1 \) is the worst possible winning coalition size whenever \( N \geq 1/g \).

**PROOF**: Lemma 3 shows that the worst possible coalition size is either \( M = 1 \) or \( M = N \). Equation (22) shows that \( W(N) > 0 \), and that \( W(1) \leq 0 \) whenever \( N \geq 1/g \).

Suppose one considers the following 4 decision–making norms: universalist (\( M = N \)), majoritarian (\( M \) is the smallest integer larger than \( N/2 \)), strict seniority (\( M = 1 \)), and libertarian (no distributinal public expenditure at all), then equation (22) implies the following ranking of alternatives, if the productivity \( g \) of government expenditure is very high.

**PROPOSITION 5**: If \( g > 0.5 \), if \( N \geq 2 \) and if \( N > \frac{2}{2g-1} \), then the 4 decision–making norms can be ranked in the following order: universalist better than libertarian better than majoritarian better than strict seniority.

**PROOF**: If \( g > 0.5 \), then if \( N \) is larger than \( \frac{2}{2g-1} \), \( N/2 + 1 \) is less than \( gN \), which (from Proposition 4) implies that \( W(N/2 + 1) < 0 \), making the libertarian outcome better than the majoritarian. The corollary to Proposition 4 shows that strict seniority is the worst of the 4 outcomes, and equation (22) shows that \( W(N) > 0 \) whenever \( g < 1 \), so that universalism is better than libertarianism.

When \( g < 0.5 \), it still is true that strict seniority is the worst option of all (if \( N \geq 1/g \)). But now the majoritarian norm is better than libertarianism (again, provided that the number \( N \) of
districts is large enough for integer problems not to get in the way). However, universality may be better or worse than majoritarianism.

Figure 1 depicts $W(M)$, when $N = 101$ and when $g = 0.3$. The four curves correspond to 4 different levels of $\gamma$. Also shown, as a dotted line, is the maximum possible level of social welfare, obtained when $G_i = G^*$ in one district and when $G_i = 0$ in every other district.

All four curves cross the horizontal axis at the same point, as equation (22) shows. That equation shows that the relative ranking of libertarian and majoritarian norms depends only on $g$, if the integer constraint is ignored. Since $g < 0.5$ in the figure, a majoritarian norm yields a positive value for $W(M)$. In figure 4, increasing the scale economy measure $\gamma$ lowers the absolute value of $W(M)$. This relation is implied, in general, from equations (9) and (22).
Now $\gamma$ itself is defined as a function of $g$, $a$, and $\rho$. So an increase in $\gamma$, holding constant $g$, can be achieved either by an increase in $\rho$, the degree to which districts' outputs are substitutes for each other, or in the labour productivity exponent $a$. The parameter $A$ also will depend on $a$ (if $\bar{L} \neq 1$). In figure 1, $A$ is held equal to 1 in each case. So the experiment is to increase $a$ or $\rho$, but in such a manner as to hold constant the value of output if all public investment were concentrated in a single district. The figure shows that increasing $\gamma$ in this manner would not affect the net value of output in a first–best world. But it will always decrease this net value when spending is decided by a legislature, with a winning coalition of size $gN$ or greater.

Figure 2 provides a slightly more detailed look at the effect of changing the coalition size $M$, this time when $g = 0.3$, $N = 101$ and $\gamma = 5$. It shows that, even though here the hypothesis of
Proposition 5 does not hold, universality in the legislature is still better than a majoritarian norm. Although \( W(M) \) is single-peaked, the peak is close to \( M = N \), and the drop-off from the peak at \( M^* \) to \( M = N \) is slight. In other words, in this simple model, increasing returns effect very little change on the conclusions of Shepsle and Weingast (1981).

4. Getting the Proposal Approved

The model presented in the previous two sections contrasts with most of the (American) literature on decision-making in legislatures, by giving the same group of \( M \) legislators both the power to propose legislation, and the power to pass the legislation. In the literature, these two groups are usually assumed distinct. A common example is a legislature in which one person proposes a bill, but in which a simple majority is required to pass the bill. In much of that literature, considerable attention is paid to the rules of the legislative process, whether (and how) bills may be amended after being proposed, and how the power to propose legislation may rotate among legislators.

Here, instead, a very simple model will be presented, the simplest possible model in which proposing legislation and voting for the legislation are done by different (possibly overlapping) groups. So from now on, the \( N \)-member legislation will have two distinguished groups: a group of \( P \) legislators gets to propose legislation, and this legislation must be approved by an additional \( L \) legislators in order to pass. In most of the literature, then, \( P = 1 \), and \( L = (N - 1)/2 \). I will refer to these two groups as consisting of “proposers” and “approvers” respectively, with the remaining \( N - P - L \) legislators “outsiders.”

Here \( P \) need not equal 1. In fact, the main goal of the next few sections of the paper is to examine how changes in the number of proposers affects the magnitude and distribution of expenditure. The weakening of the power of committee chairs in the United States Congress could be viewed here as an increase in \( P \), from 1 to perhaps the number of members of the relevant congressional committee, or the number of majority-party members of the committee. The difference from the model of sections 2 and 3 is that now the proposal cannot be dictated by some subset of the committee; it must be approved by a majority in Congress.

As before, I assume that the amount of public expenditure is the same in each of the \( P \) districts represented by proposers. As before, this may be justified by bargaining within the committee before they introduce any legislation. I will again denote this level of spending by \( G \). Now, however, they must get the approval of other legislators. This must entail additional public expenditure in these other legislators’ districts. I denote by \( H \) the level of spending in each of the \( L \) approvers’ districts.

Recall that spending in some district decreases the payoffs to residents of any other district in two ways: it raises their tax bill, and it lowers the payoff to any public expenditure in their own district. Therefore, the \( P \) proposers will endeavour to do as little spending in approvers’ districts as possible. They also will spread the spending evenly among the \( L \) proposers’ districts. (If
proposed spending were higher in district $i$ than in district $j$, then district $i$ gets a higher payoff from the legislation. If district $j$’s representative is willing to support the legislation, then district $i$’s representative must be more than willing: spending in her district could be cut somewhat without jeopardizing her support.

The problem faced by the proposers, then, is to choose levels of public expenditure $G$ for each of their own districts, and $H$ for each of $L$ approvers’ districts, to maximize their own payoffs, subject to approval by the $L$ other districts’ representatives. Since the status quo is assumed to be no public expenditure at all, then the $L$ approvers can be given a take–it–or–leave–it offer: as long as they get non–negative payoffs from the legislation, they will be willing to support it.

Therefore, the approval constraint faced by the proposers is

$$A(PG + LH)^{g/\gamma - 1}H^\gamma \geq \frac{PG + LH}{N}$$

Before proceeding, a little more notation:

$$q \equiv \frac{G}{H}$$

$$K \equiv P + L$$

So $K$ is actually the required majority for approval. The main question examined here is the effect of increasing the size of the proposing group, holding constant the required majority $K$ — that is, making some approvers into proposers. However, some calculations will be done for the effect of changing $P$ holding $L$ constant, and of changing $L$ holding $P$ constant.

Of course, the proposers will take their approval constraint as binding. Their problem can thus be regarded as the maximization of

$$A[q^\gamma P + L]^g[q^{\gamma - 1}H^g - \frac{H}{N}[qP + L]]$$

subject to the constraint

$$A[q^\gamma P + L]^g[q^{\gamma - 1}H^g - \frac{H}{N}[qP + L]] = 0$$

leading to the Lagrangian

$$\mathcal{L}(q, H, \mu) = A[q^\gamma P + L]^g[q^{\gamma - 1}H^g[q^\gamma + \mu] - \frac{H}{N}[qP + L](1 + \mu)]$$

where $\mu$ denotes the non–negative multiplier on the constraint (23). Using that constraint, the first–order condition with respect to $q$ can be written

$$\frac{H}{N}qP + L \left[ q^\gamma - (\gamma - g) \frac{q^\gamma P}{q^\gamma P + L} (q^\gamma + \mu) \right] - \frac{H}{N}P(1 + \mu) = 0$$

The first–order condition with respect to the level of spending $H$ in the approvers’ districts is
\[ gA[q^\gamma P + L]^{\gamma - 1}H^{-1}(q^\gamma + \mu) - \frac{1 + \mu}{N} (qP + L) = 0 \] (26)

Substituting from the constraint (23) into (26) yields

\[ g(q^\gamma + \mu) = 1 + \mu \]

so that

\[ \mu = \frac{q^\gamma g - 1}{1 - g} \] (27)

Equation (27) confirms that more spending is done in proposers’ districts than in approvers’, since \( q \) must be greater than 1 if the multiplier \( \mu \) is to be positive.

Substituting for \( \mu \) from equation (27) in equation (25) yields

\[ (qP + L)[\gamma q^\gamma (1 - g) - (\gamma - g)] - \frac{q^\gamma P}{q^\gamma P + L} (q^\gamma - 1) - Pqg(q^\gamma - 1) = 0 \] (28)

The overall number of districts \( N \) does not appear in equation (28) : the equation defines the ratio \( q \) of spending in proposers and approvers’ districts as a function of the ratio \( P/L \) of proposers and approvers.

Differentiation of (28) shows that \( q \) is a decreasing function of \( P/L \).

5. Proposers and Approvers : Constant Returns to Scale

While the focus of the paper is on the role of scale economies, which make dispersion of distributive expenditure among districts wasteful, I turn next to the special case of constant returns, in which \( \gamma = 1 \). In this case, the expenditure plan which maximizes overall net welfare is not unique : what is needed is for the total expenditure in all districts to equal \( A^{1/g} \). More generally, total social welfare depends only on the level \( (qP + L)H \) of total public expenditure in all districts, not on how it is distributed among the districts. However, the constrained optimum problem (24) faced by proposers does have a well–defined solution.

When \( \gamma = 1 \), equation (28) reduces to

\[ qP + L = \frac{K}{g} \] (29)

So increasing the number of proposers, while keeping constant the number \( K \) of representatives whose approval is required, has no effect on \( qP + L = (q - 1)P + K \) : the decrease in \( q \) just offsets the increase in \( P \).

When \( \gamma = 1 \), the constraint (23) that outsiders be willing to support the proposal becomes

\[ AN(qP + L)^{g - 2} - H^{1-g} = 0 \] (30)
so that changes in $P$ have no effect on the level of spending $H$ in approvers’ districts.

Total public expenditures are $(qP + L)H$, and so are unaffected by a change in $P$. Therefore, taxes paid in each district are unaffected: the decrease in spending in each proposer district exactly offsets the increase in the number of proposers’ districts.

Adding up net benefits in all districts, the total net social welfare from a policy $(q, H)$ is

$$A(q^\gamma P + L)^{\gamma/\gamma}H^\gamma / (qP + L)H$$

(The first term above is the sum of the return to public investment in each of the $K$ districts, and the second term is the total cost of the public investment.) When $\gamma = 1$, this expression does not vary with $P$ (holding $K$ constant).

Therefore

**PROPOSITION 6**: Under constant returns to scale, increasing the size $P$ of the group which proposes legislation, holding constant the majority $K$ required for approval, has no effect on total public expenditure, on expenditure in any of the $L$ districts represented by approvers, or on total social welfare. Spending in each proposer’s district, and the net payoff to each proposer, decrease, but aggregate spending in all proposers’ districts, and aggregate payoffs to all proposers, do not change.

As a yet more special case, suppose that legislation requires unanimous approval (or the legislature has a universalist norm). Given the constant returns to scale, if $P = K = N$, then the outcome would be efficient: there are no non-voting districts on which to pass the costs, and constant returns imply no problems with excessive dispersion of projects. Proposition 6 says that the outcome would still be efficient when $\gamma = 1$ and $K = N$ even if a small group of legislators got to propose the expenditure plan. The distribution of benefits shifts: but the costs of expanding the size of the proposing committee are born entirely by existing members of the committee.

Not surprisingly, this Proposition contrasts sharply with the world of sections 2 and 3. There, when the proposers of legislation did not need approval, any increase in their number would increase social welfare if $\gamma = 1$, since it reduced the extent to which costs would be shifted to other districts.

Of course, Proposition 6 refers only to the experiment of changing the size of the proposing coalition, while holding constant the majority required for approval. From equation (29), any expansion of the required majority $K$ must increasing $qP + L$. Equation (30) then implies that $H$ must decline with any increase in $K$, and further, that total expenditure $H(qP + L)$ must also fall. Further, this decrease in total expenditure must increase total social welfare.

All of these results follow from those of section 2. Proposition 6 shows that it can be assumed that $P = K$, in looking at measures of aggregate spending and welfare, when $\gamma = 1$. So the effects of changes in $K$ here are just the same as the effects of increasing $M$ in section 2. There it was shown that increasing the coalition size would decrease aggregate expenditure, and increase total social welfare.
The optimal \( q \) and \( H \) which solve the constrained optimization problem (24) are continuous in the scale parameter \( \gamma \). Therefore, some of the results of the previous section still apply if scale economies are not too large.

**PROPOSITION 7**: If \( \gamma \) is greater than 1, but sufficiently close to 1, then changing \( P \) or \( L \) will have the following qualitative effects, if the total \( K = P + L \) is not treated as constant:

1. \( \frac{\partial H}{\partial P} < 0 \);
2. \( \frac{\partial H}{\partial L} > 0 \);
3. \( \frac{\partial [qP + L]}{\partial P} < 0 \);
4. \( \frac{\partial [qP + L]}{\partial L} < 0 \);
5. Overall social welfare falls if either \( P \) or \( L \) increases.

**PROOF**: Equations (29) and (30) show that parts i and iii of the Proposition hold when \( \gamma = 1 \). By continuity, then, they hold if \( \gamma \) is larger than, but sufficiently close to, 1. Part ii follows from differentiation of equation (28), and thus is true whenever \( \gamma > 1 \), even if \( \gamma \) is very large. Total social welfare is \( (q^\gamma P + L)^{\gamma/\gamma} H^\gamma - (qP + L) \), which equals

\[
[H(q^\gamma P + L)^\gamma (q^\gamma P + L)^{\gamma/(1-\gamma)}] - (qP + L) \tag{31}
\]

When \( \gamma \) is close to 1, the first term in square brackets in expression (35) must increase with \( P \) or \( L \), from part iii of the Proposition; when \( \gamma \) is close to 1, changes in \( P \) or \( L \) have an arbitrarily small impact on the second term in square brackets in (31); when \( \gamma \) is close to 1, equation (29) shows the last term in expression (31) increases with \( P \) or \( L \). Overall, then, expression (31) must be a decreasing function of \( P \) and of \( L \), if \( \gamma \) is close to 1, establishing part iv of the Proposition.

I now return to the main procedural change under consideration, increasing the number of proposers \( P \), while simultaneously decreasing the number \( L \) of approvers, so as to leave \( K \) constant.

Whether or not \( \gamma > 1 \), increasing the number of proposers will make each proposer worse off. But when \( \gamma > 1 \), a stronger result holds: the aggregate payoff to all \( P \) proposers falls as \( P \) increases.

**PROPOSITION 8**: The aggregate payoff to all \( P \) proposers is a decreasing function of \( P \), treating the total number \( K \) of proposers and approvers as constant.

**PROOF**: It will be demonstrated that the proposers’ aggregate payoff can be increased, if the number of proposers is decreased. So suppose that \( (q, H) \) is optimal for the proposers, when there are \( P \) of them, and \( L = K - P \) approvers. Now suppose that the number of proposers is reduced to \( P' < P \), with \( L \) increasing to \( K - P' \).

A feasible policy in the new environment is to pick a new \( q' \) such that...
\[(q')^\gamma P' + L' = q^\gamma P + L\]

Since \(q' > q\), it then follows that

\[q' P' + L' < qP + L\]

if \(\gamma > 1\). In order to satisfy the approval constraint (23), the spending level \(H\) in the approvers’
districts can be increased to \(H' > H\), where

\[(H')^{1-\gamma}(q' P' + L') = H^{1-\gamma}(qP + L)\]

Now the aggregate payoff to all \(K\) proposers and approvers together is

\[
[(q')^\gamma P' + L']^{\gamma/\gamma}(H')^{\gamma} - \frac{K}{N}H'(q' P' + L')
\]

which equals

\[
(H')^{\gamma}[[(q')^\gamma P' + L']^{\gamma/\gamma} - \frac{K}{N}(q' P' + L')(H')^{1-\gamma}]
\] (31)

The changes in \(q'\) and \(H'\) mean that neither term in the large square brackets in expression (31) has changed. Since \(H'\) has increased, therefore the overall payoff to all \(K\) proposers and approvers has increased. Since the payoff to each approver is constrained to equal zero, therefore the proposed policy has increased the overall payoff to the \(P'\) proposers. A feasible policy has been constructed, which increases proposers’ aggregate payoff from that received from the optimal policy \((q, H)\) in the original situation. Therefore, decreasing \(P'\) must increase the proposers’ overall payoff.

The remaining Proposition refers to the effect on overall welfare of increasing \(P\) (holding \(K\) constant). Its proof is somewhat intricate, and is deferred to an appendix. Note that Proposition 9 applies only when the number of proposers \(P\) is large, compared with the number of approvers.

**PROPOSITION 9**: If \(L\) is sufficiently small, then when \(\gamma > 1\), increasing \(P\) (and decreasing \(L\) so as to leave constant \(K\)) has the following results:

1. \((qP + L)\) increases
2. \((q^\gamma P + L)\) decreases
3. total expenditure \((qP + L)H\) increases
4. total social welfare decreases

While Proposition 9 requires that the number \(P\) of proposers be large relative to the number of approvers, its conclusions seem to hold more widely. Figure 3 graphs social welfare as a function of \(P\), for several values of \(\gamma\). (In the examples graphed here, \(g = 0.3\), \(A = 1\) and \(N = 101\), just as
in figure 1. ) The graphs show welfare as a monotonically decreasing function of $P$, even when $P$ is small.

To see why the social welfare declines with $P$, even when $P$ is small, note that total social welfare is the total payoff to all proposers, minus the costs born by the $N-K$ outsiders: approvers’ payoffs are identically zero. Proposition 8 showed that the total payoff to proposers must decline with $P$. So unless the overall level of spending falls sufficiently to offset this effect, overall welfare must decrease as $P$ increases.

Moreover, the approval constraint (23) implies that the total payoff to proposers must equal

$$P(q^\gamma - 1) \frac{H}{N} (qP + L)$$
That means that overall social welfare can be written

\[ P\pi[1 - \frac{N - K}{P(q^\gamma - 1)}] \]

where \( \pi \) is the payoff to each proposer. From Proposition 8, a sufficient condition for social welfare to fall with \( P \) is that \( P(q^\gamma - 1) \) fall as \( P \) increases. It can be shown analytically that, if \( \gamma \) is close to, but larger than, 1, \( P(q^\gamma - 1) \) must be a decreasing function of \( P \).

In the example in which \( g = 0.3 \), figure 3 shows this is the case even when \( \gamma \) is quite large. Further, figure 5 shows that overall public expenditure \( H(qP + L) \) must increase with \( P \) in these examples, showing that both proposers (in the aggregate) and outsiders are made worse off as \( P \)
7. Concluding Remarks

Like tax competition, public input competition among regions creates a need for policy coordination. When improved public infrastructure in one region attracts mobile factors from other regions, uncoordinated decision-making by regional governments will tend to lead to excessive public investment. The main point emphasized in this paper is that unitary government is not necessarily an ideal form of policy coordination. It may be easier for (somewhat) sovereign jurisdictions to make side payments to each other than for representatives of different constituencies in a legislature.
The common pool problem in legislatures arises when members of a winning coalition can pass some of the costs of distributive spending on to losers. The absence of side payments creates another problem, as members of a winning coalition can only make compensation by spending in more districts. Increasing the size of the winning coalition reduces the first problem but exacerbates the second.

The analytic results in section 3 suggest that the common pool problem is the more serious: dictatorship seems to be the least efficient form of governance. The examples calculated in that section also suggest that there is no great harm from excessively large winning coalitions. Even though the most efficient size of winning coalition is smaller than the whole legislature, the ideal size is quite large, and the loss function quite asymmetric about the ideal size.

However, the efficiency case for large governing coalitions is weakened considerably if a distinction is made between those who propose legislation, and those who vote for it. In both congressional and parliamentary practice, the important bills which get passed are proposed by a small elite group: cabinet members or members of important committees. The analysis and examples of sections 4 – 6 suggest that expanding the size of these groups may not be a good idea. The main experiment considered there was increasing the size of this proposing group, holding constant the size of the coalition which votes for the legislation. In a sense, holding constant the size of required majority holds constant the common pool externality. Even though the number of districts in which public spending is undertaken is also being held constant, increasing the size of the proposing group seems to exacerbate the problem of excessive public investment, by making less concentrated the really large public expenditure projects.

Finally, it may be interesting to provide some comparison of “centralized” legislatures with decentralized provision. Bucovetsky (2003) analyzes the Nash equilibrium when sovereign states choose public expenditure non-cooperatively. Table 1 below uses some results form that paper. The table shows the overall social welfare for several different governance structures. In each case, $g = 0.3$, $A = 1$ and $N = 101$ (as in figures 1 – 5). The four columns list different values for the scale parameter $\gamma$. The rows list several forms of government. First, the overall first-best level of welfare (which does not depend on $\gamma$) is listed for comparison. The next three rows present results from section 3: winning coalitions of size 1, 51, and 101. The next four depict results from section 6: different sizes of proposing group when approval by 51 of 101 representatives in a legislature is required. The final row lists social welfare for a Nash equilibrium when regions’ governments behave non-cooperatively.\textsuperscript{3}

The main conclusion from this table: while the efficiency case against centralized dictatorship seems quite a strong one, there is no obvious best choice of government structure among the other alternatives.

\textsuperscript{3} When $\gamma = 5$, there actually is no pure-strategy Nash equilibrium in pure strategies when regions move simultaneously. The bottom right entry actually shows social welfare in the equilibrium to a game in which the regions make their decisions sequentially.
Table 1

<table>
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<th></th>
<th>$\gamma = 1.01$</th>
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<th>$\gamma = 1.5$</th>
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<td>0.379</td>
</tr>
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</table>
Appendix

PROPOSITION 9: If $L$ is sufficiently small, then when $\gamma > 1$, increasing $P$ (and decreasing $L$ so as to leave constant $K$) has the following results:

i) $(qP + L)$ increases

ii) $(q^\gamma P + L)$ decreases

iii) total expenditure $(qP + L)H$ increases

iv) total social welfare decreases

PROOF: The proof proceeds mostly from differentiation of equation (28) with respect to $P$, near $L = 0$.

First of all, treating the total size $K$ of proposing plus approving groups as constant, equation (28) can be written

$$[(q - 1)P + K]\Omega - Pgg = 0 \quad (A1)$$

where

$$\Omega = \gamma(1 - g) \frac{q^\gamma}{q^\gamma - 1} - (\gamma - g)\frac{q^\gamma P}{(q^\gamma - 1)P + K} \quad (A2)$$

When $L = 0$ (so that $P = K$), then

$$q^\gamma = 1/g$$

and

$$\Omega = g$$

Differentiating equation (A1),

$$\frac{\partial q}{\partial P} = -(1 - g)\frac{1 + q(\gamma - g)}{\gamma^2 K} \quad (A3)$$

when $L = 0$ — where $q = g^{-1/\gamma}$.

Since $qP + L = (q - 1)P + K$, the change in $qP + L$ is

$$(q - 1) + P\frac{\partial q}{\partial P}$$

which, from equation (A3), will be positive if and only if

$$(1 + q(\gamma - g))(1 - g) < \gamma^2(q - 1)$$

when $q^\gamma = 1/g$.

So let
\[ F_1(\gamma) \equiv \gamma^2(g^{-1/\gamma} - 1) - (1 - g)(1 + g^{-1/\gamma}(\gamma - g)) \]

Part i of the Proposition holds if and only if \( F_1(\gamma) > 0 \) for \( \gamma > 1 \). \( F_1(1) = 0 \), and

\[ F_1'(\gamma) = 2\gamma(q - 1) - q(1 - g) + [\gamma^2 - (\gamma - g)(1 - g)] \frac{\partial q}{\partial \gamma} > 0 \quad (A4) \]

The last term in \( (A4) \) must be positive, since \( q \) increases with \( \gamma \), and since the expression in square brackets must be positive. The first two terms are increasing in \( \gamma \), and equal

\[ q(1 + g) - 2 > \frac{1}{g} + 1 - 2 > 0 \]

when \( \gamma = 1 \). Therefore \( F_1(1) = 0 \), \( F_1'(\gamma) > 0 \), and part i of the Proposition has been proved.

To prove part \( ii \), note that, from expression \( (A3) \),

\[ \frac{\partial}{\partial P}[(q^\gamma - 1)P + K] = \frac{1 - g}{\gamma}[1 - g - (1/\gamma - 1)] \quad (A5) \]

at \( P = K \). The expression in square brackets equals 0 at \( \gamma = 1 \), and is negative if \( \gamma > 1 \), so that \((q^\gamma - 1)P + L\) must decrease with \( P \) at \( P = K \).

The approval constraint \( (23) \) can be written

\[ AN[(q^\gamma - 1)P + L]^{g/\gamma - 1}[(q - 1)P + L]^{-g} = [H((q - 1)P + L)]^{1-g} \quad (A4) \]

The right side of \( (A3) \) is just total expenditure, taken to a positive exponent \( 1 - g \), and so total expenditure will increase with \( P \) if and only if the left side of \( (A6) \) increases with \( P \). The derivative of the left side with respect to \( P \) is proportional to

\[ -\frac{\gamma - g}{g} \frac{\partial[(q^\gamma - 1)P + L]}{\partial P} \frac{1}{(q^\gamma - 1)P + L} - g \frac{\partial[(q - 1)P + L]}{\partial P} \frac{1}{(q - 1)P + L} \]

which (from equation \( (A3) \)) is proportional to

\[ g(1 - g^{1/\gamma}) - \frac{g^{1/\gamma}}{\gamma}[1 - g] \]

This expression can be written

\[ F_3(x) \equiv g(1 - g^x) - xg^x(1 - g) \quad (A7) \]

by setting \( x \equiv 1/\gamma \).

\[ F_3'(x) = -g^x[g\ln g + (1 - g) + x(1 - g)\ln g] \quad (A8) \]
For any \( g \in (0,1) \) the expression in square brackets in (A8) must be negative at \( x = 1 \). ( \( 1 - g + \ln g < 0 \) for all \( 0 < g < 1 \).) The expression in square brackets is also decreasing in \( x \). Then means that there is at most one \( x \) in \( (0,1) \) for which \( F3'(x) = 0 \), and such an \( x \) must be a local minimum for \( F3(x) \). From (A7), \( F3(0) = 0 \). Therefore, \( F3(x) < 0 \) for all \( x \) in \( (0,1) \), meaning that the both sides of equation (A4) must increase with \( P \), establishing part iii of the Proposition.

Finally, the overall social welfare consists of the welfare of the three groups: proposers, approvers, and outsiders. Proposition 8 showed that aggregate proposers’ welfare falls when \( P \) increases. Approvers’ welfare is identically 0. Part iii of this Proposition demonstrated that outsiders’ welfare must fall, since their welfare depends only on (and inversely with) aggregate spending, completing the proof of the Proposition.
References.


